What Economics Majors and Economists Should Know About the Supply and Demand Model: 4 Dynamics. 1/2/18

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FIRST DRAFT.
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I would value feedback on this draft.

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INTRODUCTION

This is the fourth of eight papers devoted to a critique of how the supply and demand model is taught and used in economics. This paper is concerned with the stability of the supply and demand model, and, in particular, the claim that the model shows that markets adjust smoothly and quickly to disequilibrium. I argue that the supply and demand model does not require smooth and quick adjustment to disequilibrium, and therefore that we have to impose stability on the model if it is to mirror real world economic adjustment and that there is no reason to believe that adjustment will be quick. The paper is in four sections. Section 1 reviews static stability analysis. Section 2 reviews dynamic stability analysis. Section 3 reviews the use of phase diagrams to analyses simple two-sector models. Section 4 discusses the tâtonnement process that is routinely invoked in general equilibrium models.

1 STATIC STABILITY ANALYSIS

In the 1960s and 1970s the standard intermediate textbook analysis of the supply and demand model included a discussion of static (and sometimes dynamic), stability but many textbooks now ignore this topic.[1] However, some discussion of stability is surely warranted by the fact that issues of stability are implicitly invoked in the standard supply and demand narrative when it claims that the supply and demand model has the virtue of self-correction. Further, QCS is predicated on the stability of the supply and demand model. Students

[1] Gravelle and Rees (2004) and Snyder and Nicholson (2008) both have excellent discussions of static stability.

should be aware that increasing returns to scale (IRTS) and Giffen goods can cause problems when doing QCS.[2]

The standard narrative claims first, and most importantly, that the supply and demand model is homeostatic in that it returns unguided to equilibrium when it is disturbed, and second that the adjustment to the disturbance is both rapid and smooth.[3,4] These nice properties of the supply and demand model underscore popular discussions of the efficacy of markets so the standard arguments are worth reviewing. The usual Pooh-Bah story assumes that a "shock" disturbs the equilibrium, but, as I argued in Part 1, since the supply and demand model is a deterministic model there are no error terms in the equations, and so there can be no stochastic shocks. And, as I also argued in Part 1, in an atemporal model there can be no "change" and therefore the question of whether the model when "disturbed" will return to equilibrium is, strictly speaking, meaningless.

The Pooh-Bah stability story is an example of what might be called "ball bearing economics".[5] An analogy is drawn (either explicitly or implicitly), between the behavior of a market and that of a physical system consisting of a ball bearing and a smooth steel bowl. The physical system is statically stable if, for "small" perturbations, it returns to rest at the bottom of the bowl under the forces of gravitation, air resistance, and friction. By analogy it is argued that

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[2] Marshall (1920/1961) devoted Appendix H of the *Principles* to IRTS and mentions the Giffen good case in Book III, Chapter VI, Part 4. IRTS is not compatible with perfect competition and is therefore not strictly relevant to supply and demand analysis. (Although economists did not acknowledge their defeat **Clapham won the Empty Economic Boxes debate with Pigou, see footnote 205.** The final blow was delivered in Samuelson's *Foundations*.) In what follows I do not distinguish between forward falling and backward bending supply curves.

Marshall did not introduce the Giffen good case until the third edition of the Principles. Dooley (1992) contains an interesting historical note on Marshall's interpretation of the Giffen good case. Lipsey and Rosenbluth (1971) argue that Giffen goods are likely to be more prevalent than is usually supposed.Obviously a consumer faced by a finite budget constraint cannot have a vertical demand curve over all price ranges and even less a positively sloped one. Ironically, although so much time and space is devoted to the theory of consumer behavior in microeconomics courses, ultimately we have to admit that we cannot prove, but must merely affirm, that demand curves are negatively sloped.

[3] Marshall (1920/1961 Book V, Part III, Section 6) writes: "if any accident should move the scale of production from its equilibrium position, there will *instantly* be brought into play forces tending to push it back into that position; …" (emphasis added).

[4] A static model can, at best, only address the stability of equilibrium, not the speed and path of adjustment.

[5] In the second half of the sentence quoted in the previous footnote Marshall makes an analogy between a market and a stone swinging by a string brought back to equilibrium by the force of gravity.

when disturbed P and Q return to their original equilibrium values propelled by enigmatic economic forces that overcome any market "frictions". In the standard exposition the market, like God, moves in mysterious ways.[6]

The usual economic configuration of the model, with a negatively sloped demand curve and a positively sloped supply curve, is statically stable. But, if either the demand curve, or the supply curve, or both, are perversely sloped then the model can have "equilibrium" configurations that are unstable; these cases are equivalent to balancing the ball bearing on the edge of the bowl or the top of the inverted bowl.

The standard way to do static stability analysis is to add an adjustment equation to the equilibrium condition[7]; in the Walrasian model ΔP_W has the same sign as ED_W, and in the Marshallian model ΔQ_M has the same sign as EDP_M, where ED_W is excess demand measured at some price P, and EDP_M is the Marshallian excess demand price measured at some quantity Q.

These are mechanical rules and there is no formal attempt to model the economic logic underlying them. In particular, the rules do not reflect individual agent's constrained maximization of utility or profit, and, more importantly, have nothing to say about how maximizing economic agents react to being in disequilibrium.[8] Further, as noted in Part 1, these are new models related to, but not the same as, the standard static supply and demand model.

Figure 1 illustrates the six possible cases using the usual supply and demand diagrams in (Q, P)-space.[9]

[Figure 1 goes about here.]

Panel (a) of Figure 1 shows the standard supply and demand diagram, which is statically stable in both its Walrasian and Marshallian formulations. At any price above P^e, such as P_0, there is ES and the price falls to P^e; at any price below P^e there is ED and the price increases until it reaches P^e. Therefore the Walrasian version of the model is statically stable. At any quantity above Q^e, such as Q_0, there is an EDP (WTP > WTA) and the quantity falls until equilibrium is achieved

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[6] Bengt Holstrom, the joint winner of the 2016 Nobel Prize in Economics, when asked by a reporter whether he thought CEO pay in the United States and elsewhere had become excessive replied: "It is somehow supply and demand working its magic."
http://phys.org/news/2016-10-oliver-hart-bengt-holmstrom-nobel.html

[7] This analysis assumes that the supply and demand model consists of equations (1)-(4) plus the adjustment rules – equations (69) and (70) below.

[8] Fisher (1983 11).

[9] Using the Inverse diagram makes it easier to compare the Walrasian model with the Marshallian model. These diagrams are commonplace in intermediate economics; see Boland (102 and 218).

at Q^e, and at any Q less than Q^e there is ESP (WTP < WTA) and quantity increases to Q^e. Therefore the Marshallian version of the model is also statically stable.

Panel (b) shows the case where both curves have perverse slopes. The model is unstable in both versions. Any price other than P^e is associated with movements away from equilibrium. Any quantity other than Q^e is associated with movements away from equilibrium.

Panels (c) and (d) illustrate positively sloped demand curves – the "Giffen good" case. In panel (c), where the (absolute) slope of the demand curve, $|b_{11}|$, is greater than the slope of the supply curve, d_{11}, the Walrasian model is stable but the Marshallian model is not. The reverse is true in panel (d) where $|b_{11}| < d_{11}$.

Panels (e) and (f) illustrate negatively sloped supply curves, the IRTS case. In panel (e), where the absolute value of slope of the demand curve, $|b_{11}|$, is less than the absolute value of the slope of the supply curve, $|d_{11}|$, the Walrasian model is stable but the Marshallian model is not. The reverse is true in panel (f) where $|b_{11}| > |d_{11}|$.

If only one curve is perversely sloped the Walrasian version is stable if the demand curve cuts the supply curve from below, and the Marshallian version is stable if the demand curve cuts the supply curve from above. If both curves are perversely sloped then the model is unstable in both the Walrasian and Marshallian cases.

Each of the six panels exhibits a formulation of the supply and demand model that is mathematically perfectly respectable; the equilibria exist and the adjustment process can be shown to work. However, students must be constantly warned that mathematics is not economics. Consider panels (e) and (f) of Figure 1, the IRTS cases.[10] Note that there are positive quantities supplied at zero prices. In panels (c) and (d), the Giffen good cases, there are complications because the demand curves cannot be positively sloped at "high" prices (households have finite budgets) and so the demand curves must ultimately bend back on themselves.[11] (In panel (d) there will be a second equilibrium above P^e.)

However, none of the unstable cases make sense, they all beg the question: How was the unstable "equilibrium" established in the first place? In the unstable cases the possibility that an initial equilibrium can exist is vanishingly

[10] Up to this point I have implicitly assumed that the analysis involved the so-called "short run". IRTS is a "long run" concept. But the short and long runs are merely analytical devices; see Part 2.

[11] See footnote 68.

small for the equilibrium toy model and zero for any real economy. If we start from any point close to, but not at, the purported "equilibrium", then the dynamic adjustment process always causes the system to move away from the "equilibrium". Therefore that "equilibrium" can never exist in the first place.

The ball bearing analogy to the behavior of a market with many transactors is highly questionable. Pooh-Bah's physical analogy violates the usual assumption in physics and partial equilibrium economics that we are dealing with a closed system. How does the ball bearing get to be balanced on the edge of the bowl, or how does it end up at the apex if the bowl is inverted? In physics the experimenter performs these functions under carefully controlled conditions. But what economic process brought about the initial economic equilibrium when all paths are unstable? Obviously the invisible hand! The physical ball bearing has an infinite number of unstable equilibrium states, precariously balanced on one of the points on the rim of the bowl, or sitting on the top of the inverted bowl. Economic systems can possess multiple equilibria, but they pose serious policy problems because when we are doing QCS policy exercises we need to know from which equilibrium we start and at which equilibrium do we end up. Different equilibria are likely to exhibit significantly different welfare properties.

What the student needs to take away from this exercise is: First, and crucially important, unstable equilibria cannot exist in real economies.[12] Even if the supply or demand curves are perversely sloped the slopes must satisfy the stability conditions. Indeed, economists routinely invoke the stability conditions to constrain the behaviors of economic actors. Second, the self-equilibrating property often ascribed to markets is not something that can be derived from the model that underlies the standard narrative. Finally, majors should take learn the "First Commandment of QCS": QCS is only meaningful in the stable cases whether the model is in its Walrasian or Marshallian form.[13]

[12] Although events such as the Great Depression, the Great Recession and hyperinflations clearly show that real economies may be subject to massive coordination failures, all real world economies have ultimately come to rest with positive outputs and stable rates of inflation.

[13] Samuelson (Part 2) is the canonical reference. Samuelson's Nobel Prize was awarded "for the scientific work through which he has developed static and dynamic economic theory …". http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/1970/ Samuelson's dynamics focused on price, not quantity, changes.

2 DYNAMIC STABILITY ANALYSIS[14]

Samuelson provided a mathematical argument to replace Pooh-Bah's verbal narrative[15] by writing simple differential equations – again taken from mechanics – to characterize the time trajectories of the price (Walras) or quantity (Marshall) in situations of excess demand (Walras) or excess demand price (Marshall). In our notation and using finite differences:

(1) $\Delta P_t/\Delta t = \lambda \text{ EDt} = \lambda (Q^d_t - Q^s_t)$ where $\lambda \in R$ and $\lambda > 0$

and

(2) $\Delta Q_t/\Delta t = \lambda \text{ EDP}^d_t = \lambda (P^d_t - P^s_t)$ where $\lambda \in R$ and $\lambda > 0$

As in section 6.1 these equations are added to equations (1)-(4) and since we have added an extra equation to the original supply and demand model, it is now a new model and the behavior of the new model tells us nothing about the behavior of the original model. Equation (69) generates an inhomogeneous, first order, linear difference equation with constant coefficients.[16]

(3) $\Delta P_t = \lambda \text{ EDt} = \lambda (Q^d_t - Q^s_t)$ where $\lambda \in R_{++}$ and t is an integer

$$= \lambda (d_1+d_{11}P_t - s_1 - s_{11}P_{t-1})$$

$$P_t - P_{t-1} = \lambda (d_1 - s_1) + \lambda(d_{11} - s_{11})P_{t-1}$$

$$P_t = P_{t-1}+ \lambda (d_1 - s_1) + \lambda(d_{11} - s_{11})P_{t-1}$$

(4) $P_t = \lambda (d_1 - s_1) + P_{t-1} + \lambda(d_{11} - s_{11})P_{t-1}$

$$P_t = \lambda (d_1 - s_1) + [1+\lambda(d_{11} - s_{11})]P_{t-1}$$

The general solution for P_t is:

(5) $P_t = P^e + (P_0 - P^e)[1 + \lambda(d_{11} - s_{11})]^t$ where $[1 + \lambda(d_{11} - s_{11})] \neq 1$

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[14] Fisher (1983) is the classic reference for stability analysis in both partial and general equilibrium models. See also Fisher 2011 for a beautifully succinct, verbal, review of the stability problem.

[15] Sleeman 2017a sec.1.

[16] I follow standard practice and use t as a subscript although I would prefer to use i (for iterations) since I am skeptical that this analysis has anything to do with "time" in usual meaning of that word. Attaching a t subscript to the endogenous variables and then tracing out their paths using a parameter drawn from a hat does not seem to capture the rich fabric of associations that are usually attached to the idea of time.

If $P_0 \neq P^e$, that is, the system is not initially in equilibrium, then the behavior of the model depends on $[1 + \lambda(d_{11} - s_{11})]^t$; the size of $[1 + \lambda(d_{11} - s_{11})]$ determines whether the model is stable, the sign of $[1 + \lambda(d_{11} - s_{11})]$ determines the type of time path taken by P_t.

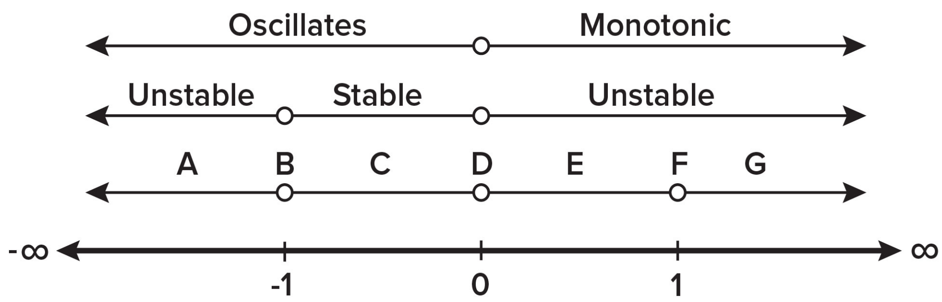
The model is dynamically stable if $-1 < |[1 + \lambda(d_{11} - s_{11})] < 1$ and $|[1 - \lambda(s_{11} - d_{11})]| \neq 0$. The model converges monotonically to P^e if $0 < [1 + \lambda(d_{11} - s_{11})] < 1$, and oscillates to equilibrium if $0 > 1 + \lambda(d_{11} - s_{11}) > -1$.

If the term in square brackets is converted to the form $1 - \lambda(s_{11} - d_{11})$, then stability requires $1 > |[1 - \lambda(s_{11} - d_{11})]| > -1$ and $|[1 - \lambda(s_{11} - d_{11})]| \neq 0$. Monotonic convergence requires that $1 - \lambda(s_{11} - d_{11}) > 0$. The model converges to equilibrium with damped oscillations if $1 - \lambda(s_{11} - d_{11}) < 0$. And so with a positively sloped supply curve and a negatively sloped demand curve the supply and demand model approaches equilibrium monotonically if $\lambda < 1/(s_{11} - d_{11})$, where $s_{11} - d_{11}$ is the familiar rate at which a price change erodes excess demand (see section 4.1).[17,18]

The schematic illustrates the possible behaviors of the difference equation.

[17] Notice that although the dynamic behavior of the model does not appear to depend on the size of excess demand when $P = 0$ $(d_1 - s_1)$, it is clear from equation (71) that it does.

[18] The supply and demand model is stable in the perverse cases so long as the absolute value of the slope of the supply curve, in (P, Q)-space, is greater than the absolute value of the slope of the demand curve.



A	$1-\lambda(s_{11}-d_{11}) < -1$	Unstable oscillations. (In practice P_t will fall to zero.)
B	$1-\lambda(s_{11}-d_{11}) = -1$	Oscillations around P^e.
C	$-1 < 1-\lambda(s_{11}-d_{11}) < 0$	Stable oscillations.
D	$1-\lambda(s_{11}-d_{11}) = 0$	P_t is a constant
E	$0 < 1-\lambda(s_{11}-d_{11}) < 1$	Stable and monotonic.
F	$1-\lambda(s_{11}-d_{11}) = 1$	P_t increases without bound
F	$1 < 1-\lambda(s_{11}-d_{11})$	Monotonically unstable. (In practice P_t will fall to zero.)

With all four supply and demand parameters constant the adjustment process depends on the size of the parameter λ. Although λ is usually referred to as the speed of adjustment, students should note that higher values of λ are not necessarily associated with more rapid convergence to equilibrium. A high λ means the buyers and sellers react more strongly to disequilibrium and this may cause the system to overshoot. It is easy to see that as λ becomes larger the model moves through the whole spectrum of stability results A to G. And, whatever the value of λ, if the system converges to P^e then it may technically require an "infinitely" long time to do so. Of course we can add a stopping rule that "equilibrium" is achieved if $\Delta P_t < \varepsilon$, where ε is an arbitrarily small constant.

If the numerical values of all five parameters were known then it would be a matter of a simple arithmetical calculation to determine which of the many possible paths the model would follow, but, as I argue in Part 9, this information is unlikely to be available to us. In the absence of empirical estimates of the parameters it is not possible to prove that the standard Pooh-Bah claim of (rapid) monotonic convergence to equilibrium is in any way a preferred motion of the system. Without numerical values for the parameters we are left, analytically, with a taxonomy of possible dynamic behaviors of the model. *A priori* there is no reason to assume that the supply and demand model converges monotonically to a stable equilibrium even in its standard configuration. We must therefore impose stability and monotonicity on the model on the grounds, invoked in the previous section, that it is implausible to think that real world markets can be unstable.

Equations (1) and (2) were designed to describe the behavior of inanimate physical objects. But economic agents and economic markets are not like planets following elliptical paths impelled by an inverse square law, or like ball bearings rolling in a steel bowl subject to friction, air resistance, and gravity. Samuelson's mathematics may be relatively sophisticated, but the economics is not; there is no discussion of what economic forces are at work to drive the supply and demand model. Samuelson's approach results in a taxonomy of possible price trajectories, not an explanation of how actual markets come to behave in the ways that they do; a taxonomy that is not determined by the maximization of the economic agents' objective functions subject to the constraints they confront, even though it was Samuelson who is usually identified with the post–World War II conversion of economists to the idea that the foundation stone on which economics is built is constrained extremization by economic agents.[19] Further the speed of adjustment coefficient, λ, has no economic interpretation; λ is just a number pulled out of a hat. There is no attempt to provide an economic explanation of why one market has a low "speed of adjustment", while another has a high "speed of adjustment".[20]

3 PHASE DIAGRAMS

Turning to the two-good case we can use equations (28)-(29) and (31)-(32) to derive equations that specify the combinations of P^e_1 and P^e_2 for which excess demand in market 1 is zero (ME1), and excess demand in market 2 is zero (ME2).[21]

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[20] Students should note that although physicists and engineers routinely check that the items on both sides of their equations have the same dimensions, economists, especially when teaching supply and demand, usually pay no attention to dimensional analysis. In equation (69) the variables on the right hand sides of the equations are quantities measured in pounds, tons, gallons, barrels, boxes, numbers of cars, or some such unit. The price variables on the left hand sides of the equations are measured in some nominal monetary unit, such as dollars or thousands of dollars. Therefore if d_1 and s_1 are in, say, tons and if P is in, say, dollars then $d_{11} = \Delta Q^d/\Delta P$ and $s_{11} = \Delta Q^s/\Delta P$ must have dimensions tons/\$. Hence, the λs in equations (69) and (71)-(73) must have dimensions \$/ton.

[21] See Hands (2010).

(74) ME1: $P^e_2 = -\left(\frac{d_1 - s_1}{d_{12}}\right) + \frac{s_{11} - d_{11}}{d_{12}} P^e_1$

(75) ME2: $P^e_2 = \frac{d_2 - s_2}{s_{22} - d_{22}} + \frac{d_{12}}{s_{22} - d_{22}} P^e_1$

In my experience students often have problems with phase diagrams. It is therefore wise to begin by discussing Figure 1. The left hand panel, is about market 1, and shows what happens to P_1 when P_2 changes. The ME1 line is the locus of all P_1 and P_2 price combinations for which market 1 is in equilibrium. Choose an arbitrary point on ME1 and draw lines to mark the $P^e{}_1$ and $P^e{}_2$ equilibria. Holding $P^e{}_1$ constant allow P_2 to rise to the point A/L (above/left).

[Figure 1 goes about here.]

Since P_2 has increased and because 2 is a substitute for 1 the demand for 1 will increase. The increase in demand for good 1 causes excess demand for 1 at the price $P^e{}_1$. According to the adjustment rule P_1 will increase so long as there is excess demand. Therefore, any point above, and to the left of, ME1 is associated with an increase in P_1. The same argument to show that at any point like B/R (below/right) the lower P_2 will cause the demand for 1 to decrease, and the subsequent excess supply will cause P_1 to fall according to the adjustment rule.

Next move to the right-hand panel and emphasize that it is about market 2, and shows what happens to P_2 when P_1 changes. The ME2 line is the locus of all P_2 and P_1 price combinations for which market 2 is in equilibrium. Choose an arbitrary point on ME2 and draw lines to mark the $P^e{}_1$ and $P^e{}_2$ equilibria. Holding $P^e{}_2$ constant allows P_1 to decrease to A/L. Since P_1 has decreased and because 1 is a substitute for 2 the demand for 2 will decrease. The decrease in demand for good 2 causes excess supply of 2 at the price $P^e{}_2$. According to the adjustment rule, P_2 will decrease as long as there is excess supply. Therefore any point above and to the left of ME2 is associated with a decrease in P_2. The same argument shows that at any point like B/R the higher P_1 will cause the demand for 2 to increase and the subsequent excess demand will cause P_2 to increase according to the adjustment rule.

Only when students have completely mastered Figure 1 should we discuss Figure 2. In Figure 2 ME1 and ME2 are shown on the same diagram. If the two commodities are substitutes, the ME1curve cuts the ME2 curve from below. For a solution in the positive orthant we require two conditions to hold: (1) the standard coefficient restrictions that ensure that the vertical intercept of the ME2 curves will be higher than the vertical intercept of the ME1 curves, and (2) the dominant diagonal assumption that the (absolute) values of the slopes of the ME2 curves must be greater than the (absolute) values of the slopes of the ME1 curves; that is $(s_{11}-d_{11})(s_{22}-d_{22}) > d_{12}d_{21}$.

The left

[Figure 2 goes about here.]

hand panel illustrates the case where goods 1 and 2 are substitutes. In the affine case there is a unique two-market equilibrium at $P^e{}_1$ and $P^e{}_2$. In sector (a) there is excess demand for good 1 and excess supply of good 2. Prices can move due east, due south, or southeast (P_1 must always increase and P_2 must always decrease). Therefore, starting from any arbitrary combination of P_1 and P_2 different from their equilibrium values, P_1 and P_2 must approach those equilibrium values as "time" increases.

If the price trajectory strays into sector (b), or if we start from any arbitrary point in (b), then P_1 and P_2 must both decease because both goods are in excess supply in sector (b). If the price trajectory strays into sector (c), or if we start from any arbitrary point in (c), then P_1 will decrease because good 1 is in excess supply in sector (c). But in sector (c) there is excess demand for good 2, so its price will rise; hence the price trajectory must follow a path that moves both prices closer to equilibrium. If the price trajectory strays into sector (d) or if we start from any arbitrary point in (d), then P_1 will decrease because there is excess demand for good 1 in (d). But there is also excess demand for good 2, and so its price will also rise. Therefore, the price trajectory must follow a path that moves both prices closer to equilibrium.

Of course, we know nothing about the actual paths of the prices or their speed of convergence; it might take an infinitely long time for prices to achieve equilibrium or the system may simply fail to do so at all. But we can show, as is well known (see Scarf (1960) and Gale (1963)), that even the two-market toy model is stable only when the goods are substitutes.

As long as the system is stable the diagram can be used to do QCS exercises, changing the positions and/or the slopes of one or both curves and observing what happens to the two prices. However, what happens to the two quantities has to be deduced from the equilibrium equations.

Majors should be aware that any model in which at least one market is unstable is not worth analyzing, because there could never be a viable equilibrium. They should also be reminded that these sorts of supply and demand exercises are Pooh-Bah economics. The supply and demand model is a static model, it is always in equilibrium; we are always at the intersection of the two ME curves.

4 THE TÂTONNEMENT PROCESS

Do we want to reveal to our majors, what in all honesty we should reveal, that price adjustment in economics is usually conceived as a tâtonnement process? (There are, of course, many versions of the tâtonnement process.) Section 5.1 discussed static stability analysis as a set of adjustment rules. How are these rules implemented? This question has bedeviled economics at least since the mid-1870s when Walras, engaged in the immense task of constructing from scratch a full-scale general equilibrium model and with greater ambitions than simply finding an equilibrium set of prices,[22] essentially evaded the issue by invoking an "auctioneer" to reset out-of-equilibrium prices, one by one, until transactions could take place in equilibrium and with no transactions allowed until an equilibrium set of prices was found.

If we ask students to study the tâtonnement process, we should at least replace the venerable auctioneer with something that twenty-first century students can more readily identify with, an internet-based information system (run by the government and financed by a per capita tax on buyers and sellers). The computer system, which I will call Tinkerbell, can collect, aggregate, and compare offers and bids, and is programmed to adjust prices according to the rules $\Delta P = \lambda(ED)$ (in the Walrasian model), and $\Delta Q = \lambda(EDP)$ (in the Marshallian model) where $\lambda \in R$, and $\lambda > 0$ if there is excess demand (price).

Stupidity has been defined as doing the same thing over and over again and expecting a different outcome. By this standard, if we admit the unstable market cases, Tinkerbell is decidedly stupid, more stupid than a human auctioneer would be because the auctioneer, unlike Tinkerbell, can learn from experience. In the unstable cases Tinkerbell continues to raise prices in the face of excess demand, and continues to lower prices in the face of excess supply, even though these adjustments exacerbate the excess demand or excess supply, moving the model in the wrong direction. This is equivalent to entering a freeway against the oncoming traffic and proceeding to accelerate faster and faster, stubbornly ignoring the evidence that you are going in the wrong direction. Tinkerbell is a poorly programmed computer; she is incapable of changing her behavior in the face of clear evidence that what she is doing will lead to disaster.

[22] The *Élements* are much closer in scope and ambition to the program pursued by the Classical economists than to the narrow focus of the marginalists.

However, it is possible to reprogram Tinkerbell to learn from her mistakes; the new program rules are $\Delta P = \lambda(ED)$ etc. with $\lambda > 0$ if $ED_1 > 0$ and if $ED_1 < ED_0$ but set $\lambda < 0$ if $ED_1 < 0$ when $0 > |ED_1| > |ED_0|$; reverse the rule if $ED_1 < 0$.[23]

In the simple toy model with its affine supply and demand functions, Tinkerbell can also be programmed to determine the equilibrium price simply by using two price-quantity points to estimate the supply and demand curves. If we take the algebra seriously we see that the correct λ coefficient in the adjustment equations can be deduced from the slopes of the supply and demand curves.

An interesting issue is how does Tinkerbell know when a supply or demand curve has shifted? Does the program monitor the market continually – as our operating systems monitor our computers to see if we have made a keyboard stroke – even after an equilibrium has been established? Unless we confine the analysis to steady states Tinkerbell has to be eternally vigilant to sense every shift in supplies or demands (although this is hardly important in a static model). If all variables are represented by real numbers, Tinkerbell has to be infinitely discriminating or programmed to ignore minute changes.

Although Tinkerbell may be able to do calculations at the speed of light, the consumers and producers who respond to her requests for information will do so at more prosaic speeds and the "time" to achieve equilibrium may be non-trivial. In a world in which the only information transactors have is the price they transmitted to Tinkerbell, Tinkerbell would need an algorithm to match the millions of buyers with the millions of sellers with their different quantities demanded and supplied, so that they would know with whom to transact.

Unfortunately, not only is the tâtonnement process unrealistic it is also, as already noted, unstable (see footnote 71).

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[23] This is equivalent, in classical control terms, to adding a derivative element to the standard proportional control. Of course the subscript refers to successive iterations, Tinkerbell 1, Tinkerbell 2, etc., not to time.

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FIGURES

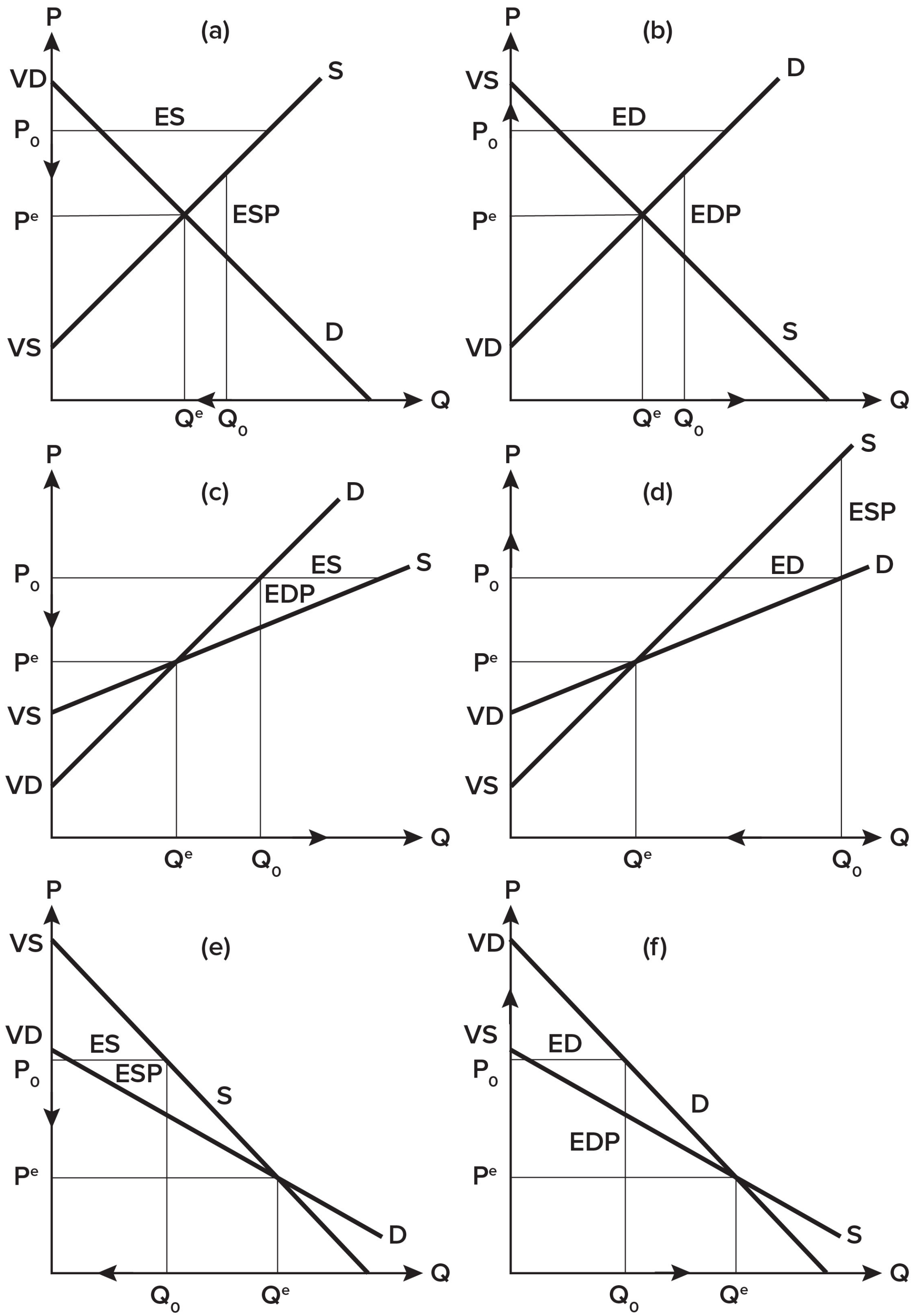

Figure 1

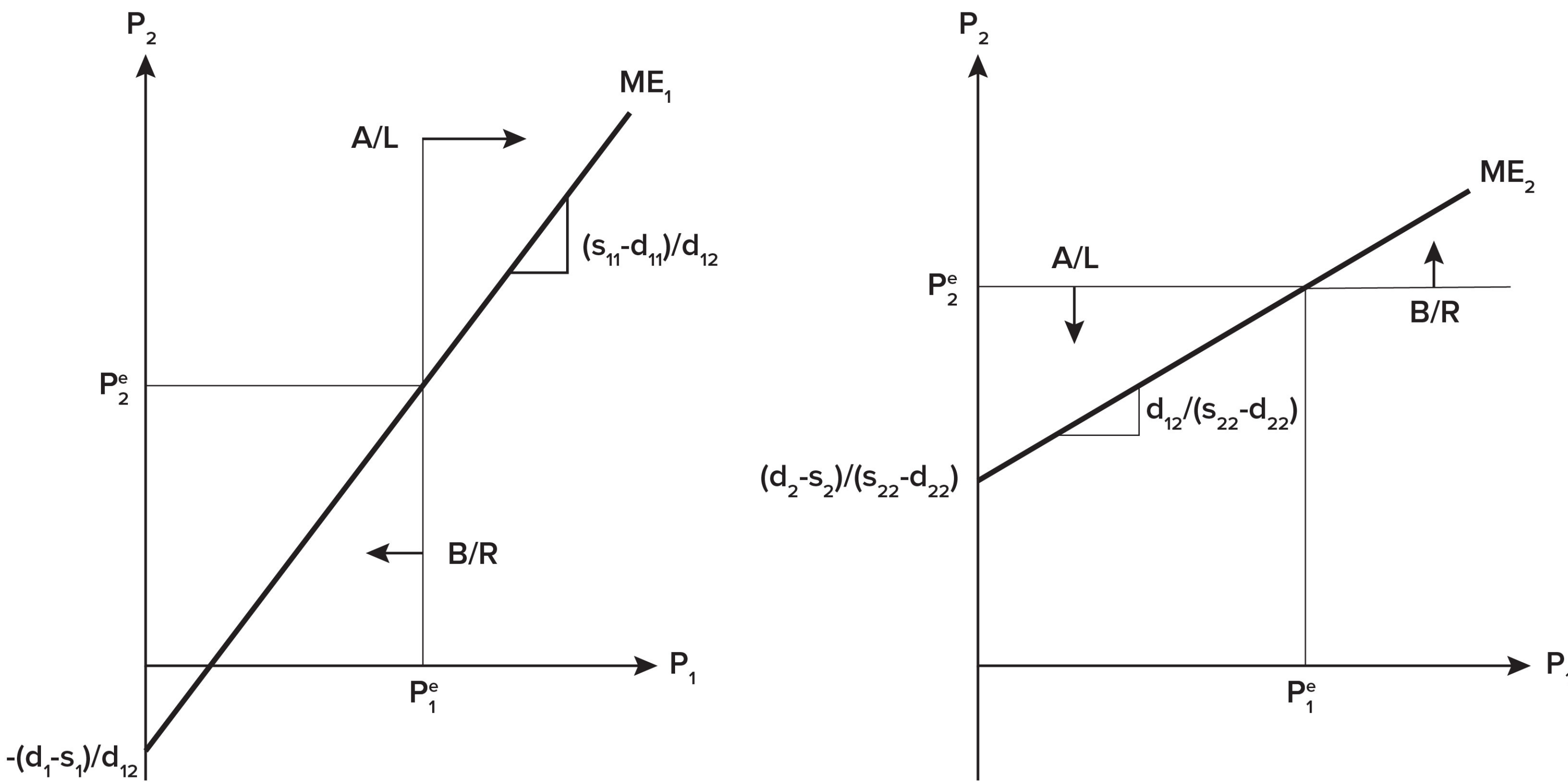
P_2
ME_1
A/L
$(s_{11}-d_{11})/d_{12}$
P^e_2
B/R
P_1
P^e_1
$-(d_1-s_1)/d_{12}$
P_2
ME_2
A/L
P^e_2
B/R
$d_{12}/(s_{22}-d_{22})$
$(d_2-s_2)/(s_{22}-d_{22})$
P_1
P^e_1

Figure 2

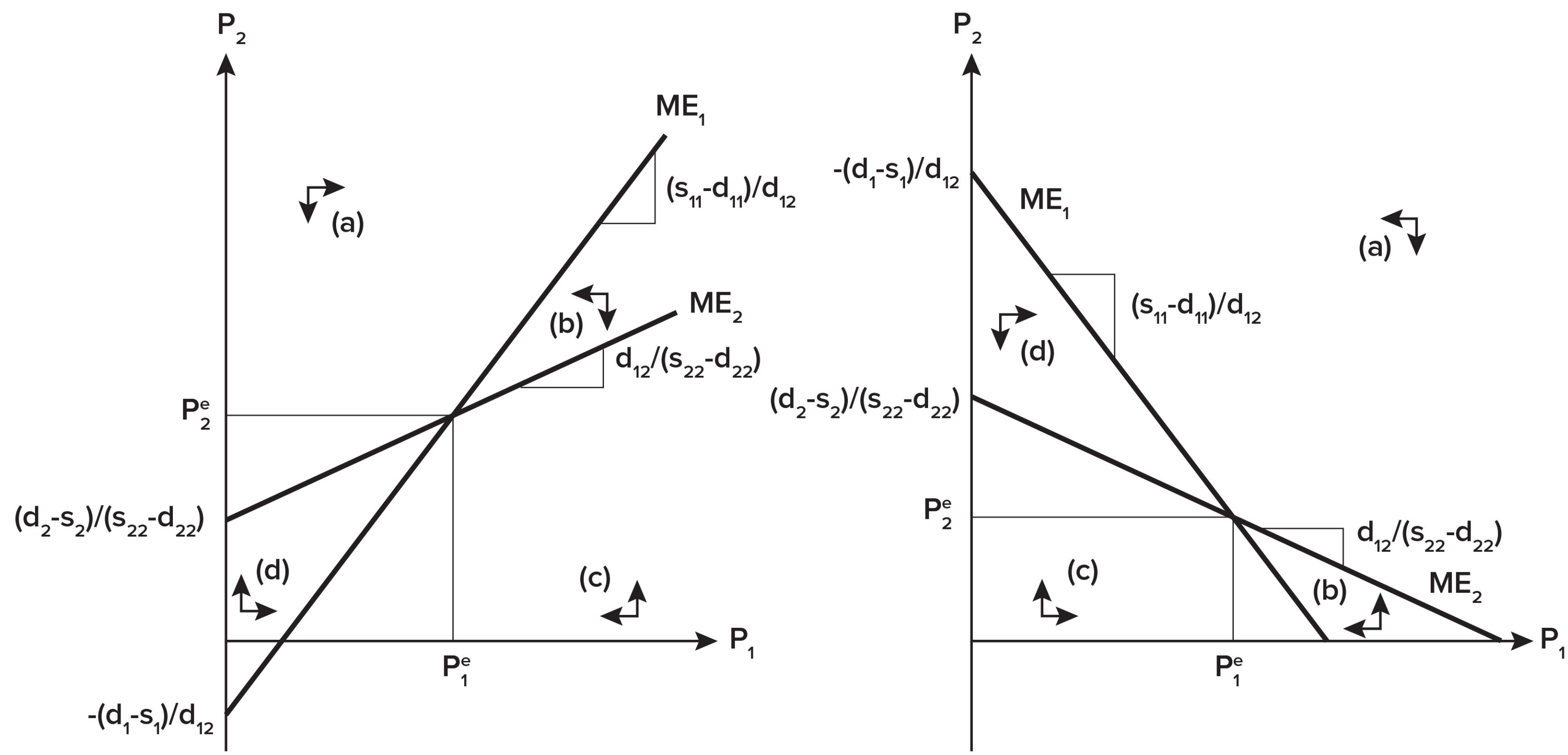
P_2
ME_1
$(s_{11}-d_{11})/d_{12}$
(a)
ME_2
(b)
$d_{12}/(s_{22}-d_{22})$
P^e_2
$(d_2-s_2)/(s_{22}-d_{22})$
(d)
(c)
P_1
P^e_1
$-(d_1-s_1)/d_{12}$
P_2
$-(d_1-s_1)/d_{12}$
ME_1
(a)
$(s_{11}-d_{11})/d_{12}$
(d)
$(d_2-s_2)/(s_{22}-d_{22})$
P^e_2
$d_{12}/(s_{22}-d_{22})$
(c)
(b)
ME_2
P_1
P^e_1

Figure 3