CM17: ELASTICITY (2/20/19)

MOST, BUT NOT ALL, OF WHAT YOU NEED TO KNOW

1. What is an elasticity?

2. Why do we use percentage changes when calculating elasticities?

3. How is PED defined?

4. Why is a PED a negative number? What convention do we adopt in order to avoid dealing with negative numbers?

5. How are PEDs classified in terms of their numerical values?

6. How, very roughly, is PED related to the slope of the demand curve?

7. How is PED related to what happens to total revenue (TR = PxQ) when price changes? Why is this important to a government when deciding where to apply sales taxes?

8. What determines PED?

9. What do PES, IED, and CED stand for?

1. EMPIRICAL ECONOMICS

1. Economic theory, like all theory, is qualitative – it tells us which variables cause X to increase or decrease, but it does not specify by how much X changes when those variables change. On the few occasions in this course that I have used numbers they were purely illustrative, I made them up. But if economics is to tell us interesting things about how the economy works, especially if economists are going to be able to give useful policy advice, then economists must add quantitative content to their models. I have emphasized in earlier lectures that economists are largely concerned with doing what I dubbed Empirical Economics; that is, collecting and statistically analyzing data.

2. Empirical work in economics is very difficult; economists often have only small data sets (small samples), and when economists do have large data sets the observations are not likely to be drawn from the same statistical population because the economy is an evolving system that continually adapts to

circumstances, such as the various oil shocks that we have experienced since 1973. The data economists use is "dirty", it is subject to a lot of measurement error[1], and the variables that we can observe are often different from the variables that we would like to observe; for example, economists have data on how many bananas were actually bought and sold, not the quantities demanded and supplied, and we have data on years of schooling, but not on how much was learned. Economic data seldom meet the requirements for the application of classical statistical methods and so economists have developed their own sub-discipline, econometrics, which attempts to develop techniques for handling the many statistical problems that arise when doing empirical work with economic data.

2. RESPONSIVENESS AND SLOPE

1. One question economists would like to answer is how responsive are the quantities demanded and supplied of gas to changes in the determinants of the quantities demanded and supplied of gas. In particular, we would like to know how responsive are the supply and demand for gas to changes in the price of gas. One obvious approach would be to try to estimate the demand and supply curves for gas, and then calculate the slopes of those curves; in the terminology we used in earlier Commentaries, we would determine how "steep" or "flat" the curves are. But the slope of a curve depends on the units in which the variables are measured.[2] We will get different slopes if we measure price in cents, or dollars, or thousands of dollars; we will get different slopes if we measure quantities in ounces, or pounds, or tons. If we get different slopes every time we use different units of measurement then the slope of a supply (demand) curve will give us different (although equivalent) measures of the responsiveness of the quantity supplied (demanded) to changes in price, depending on the units we use.

2. The slope of a straight-line is a constant. Let Y depend on X. The slope of the line relating Y to X is the "rise over the run", the change in Y over the change in X. Say the change in the price of X is $2 and the change in Y is 100 (short) tons. Then the slope of the curve is 100 (short) tons divided by $2, which is 50 (short) tons per dollar. But if we decide to measure Y in pounds, then the ratio

[1] An engineer would say that it has a low signal to noise ratio.

[2] If I am driving at 100 miles per hour then I am also driving at 528,000 feet per hour, or 0.028 miles per second, or 146.67 feet per second, or 162.9 kilometers per hour. We get different numbers, different measures of speed, if we use different units of measurement.

becomes 200,000 pounds divided by $2, which is 100,000 pounds per dollar. If we use cents instead of dollars, then the slope becomes 100 (short) tons divided by 200 cents, and the slope is now ½ a (short) ton per cent. (See Figures 1 and 2a and 2b.)[3] Notice that in each of these cases the slope was not a pure number but was expressed as a ratio, a money measure to a quantity measure.

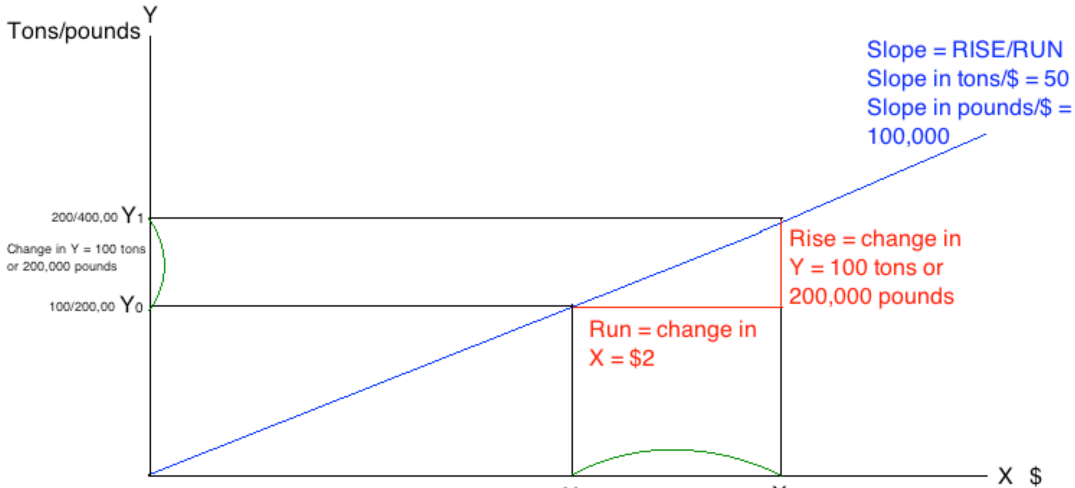

Figure 1

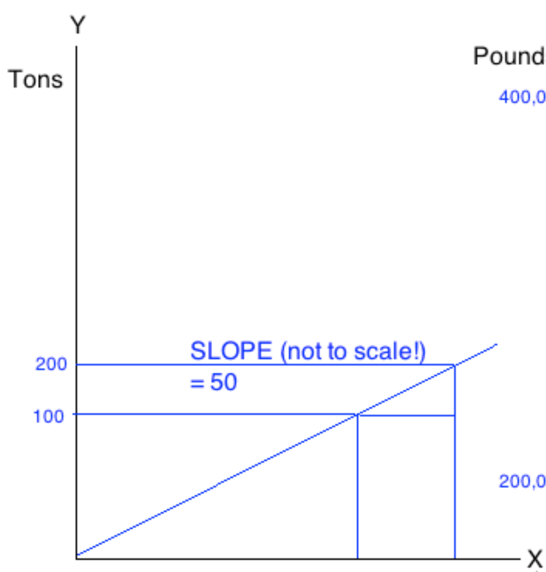

Figure 2a

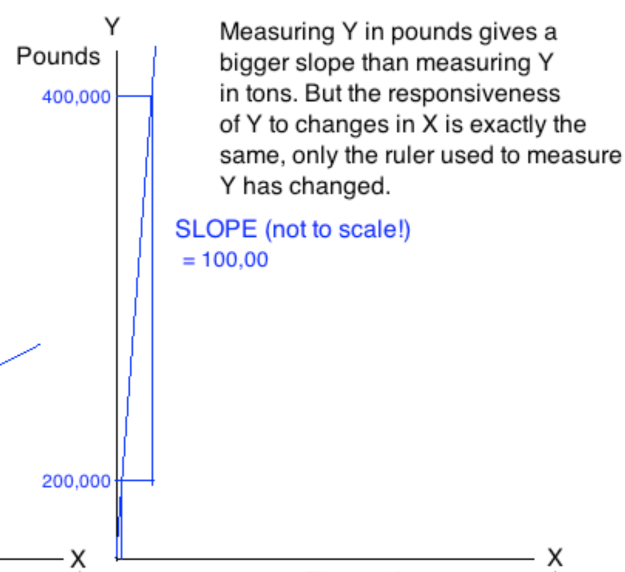

Figure 2b

[3] Notice that I have swapped the P and Q axes for this example.

3. Problems arise if we start talking about one slope being greater than another, or one curve being steeper than another curve; these statements only make sense if the variables are measured in the same units. Say we try to compare the price responsiveness of salt, with that of Honda Accords. And say that when the price of a packet of table salt falls by 5c per packet, the quantity bought and sold increases by 2million packets per year. And assume that if the price of a Honda Accord falls by $500 per car, then the number of Accords sold increases by 1.5 million cars. We can't say which commodity is more responsive to a price change because the units are different; packets of table salt are not comparable/commensurable with automobiles.

3. ELASTICITY

1. The English economist Alfred Marshall came up with a solution to these problems in his *Principles of Economics* (1890); he introduced the concept of what he called an elasticity. An elasticity is a measure of the responsiveness of a dependent variable, for example the quantity demanded of Honda Accords, to a change in an independent variable, say the price of a Honda Accord. There are six determinants of the quantity demanded (independent variables) and six determinants of the quantity supplied (independent variables); therefore there are six demand elasticities, and six supply elasticities. We will concentrate on the so-called price elasticity of demand, PED.

2. Marshall's solution to how to measure responsiveness was to work with percentage changes; if you divide a percentage change by a percentage change, then you get a pure number. One hundred miles divided by one hour is 100 m.p.h. But 6 percent divided by 2 percent is 3 not 3X; the percentages cancel.

3. The general definition of an elasticity is: *The elasticity of Y with respect to X is the percentage change in the dependent variable, Y, brought about by a given percentage change in the independent variable, X*: the percentage change in Y divided by the percentage change in X; Elasticity of Y with respect to X = $\%\Delta Y/\%\Delta X = (\Delta Y/\Delta X)(X/Y)$.[4]

[4] Formally the elasticity of Y with respect to X is $E_{YX} = [(\Delta Y/Y) \times 100]/ [(\Delta X/X)\times 100] = (\Delta Y/\Delta X)(X/Y)$, where the delta ($\Delta$) represents a change in the variable. In elementary economics we usually assume that ΔX is one unit of X. In practice economists use derivatives rather than finite differences although in the linear case they give the same result; $E_{YX}= (dY/dX)(X/Y)$.

4. PRICE ELASTICITY OF DEMAND, PED

1. *The Price Elasticity of Demand (PED) is the percentage change in the quantity demanded of X divided by the percentage change in the price of X*: that is PED_X = the percentage change in the quantity demanded divided by the percentage change in the price of X, which equals $\%\Delta Q^d{}_X / \%\Delta P_X = (\Delta Q^d{}_X / \Delta P_X)(P_X / Q^d{}_X)$[5]. Because the numerator (the top of the fraction), and the denominator (the bottom of the fraction), are both percentages, PED, like all other elasticities, is a pure number; it has no units of measurement. An elasticity is just a number, eight per cent divided by four per cent is just two (8% /4% = 2), not two something per something.

2. It is important to remember that PED_X (I will usually drop the subscript) measures the effect of the change in the price X not some other price, because there are other elasticities that deal with the prices of goods and services related to X.[6]

3. There is a technical problem that we have to deal with when talking about PED. Demand curves slope down to the right, they have negative slopes, and so when the price of X increases (+) the quantity demanded falls (–), and the ratio of a negative number to a positive number is a negative number; similarly, if the price of X decreases (–) then the quantity demanded of X will increase (+), and so the ratio is again negative. Therefore PED is a negative number.

4. For example: If the price of X falls by 5%, and this causes the quantity demanded of X to increase by 10%, then the PED will be 10% / –5% = –2, and if the price of X increases by 5% and this causes the quantity demanded of X to decrease by 10%, then the PED will be –10%/5% = –2. *PED is always a negative number*. But most people have problems with determining the relative sizes of negative numbers: Quickly, which is bigger, –½ or –2,000?[7]

We will therefore adopt a **convention**: *we will ignore the minus sign when discussing PEDs*, that is, we will treat –5 as 5 and –½ as ½ and –1 as 1.[8] Luckily, it turns out that that at this level the minus sign doesn't matter.

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[5] When calculating PED we use the average of the two Ps and the average of the two Q^ds.

[6] Cross elasticities of demand measure the responsiveness of the quantity demanded to changes in the prices of substitutes and complements.

[7] –½ is 4,000 times *bigger* than –2,000.

[8] Mathematically we are really calculating the absolute value of the PED.

5. CLASSIFYING PEDs

1. The smallest possible PED is 0, which would occur if the quantity demanded was totally unresponsive to changes in the price of X – we always buy the same amount of pizza irrespective of its price, and the demand curve for pizza would therefore be a vertical straight line.[9] The largest PED would be "infinite" – which is not a number but a statement that some process continues without end, or, without bound; mathematically we are faced with the task of dividing by zero, which is an undefined mathematical operation. What an "infinite" PED means (PED = "∞") is that the demand curve is horizontal. Therefore, an infinitesimally small drop in price would cause the quantity demanded to increase "without bound", and an infinitesimally small increase in price would cause the quantity demanded to fall to zero – such "knife edge" behavior is not observed in real economies. We say that *a demand curve with a PED = 0 is perfectly inelastic* (zero response to the price change), and that *a demand curve with a PED = "∞" is perfectly elastic*.

2.We distinguish between demand curves with PED = 0 (perfectly inelastic), demand curves with PEDs between zero and 1 (inelastic), demand curves with PED = 1 (unit elastic), demand curves with PEDs between 1 and infinity (elastic), and demand curves with "infinite" PEDs (perfectly elastic). The zero, one, and "infinite" cases are only of theoretical interest (see Table 1.)

TABLE 1

PED = 0	0<PED<1	PED = 1	1 < PED < ∝	PED = ∝
"Perfectly" inelastic demand	Inelastic demand	Unit elastic demand	Elastic demand	"Perfectly" elastic demand

3. "Very roughly" speaking, inelastic demand curves have large slopes and are "steep", and elastic demand curves have small slopes and are "flat". See Figure 3 where, *at the point where the two demand curves cross*, the "flatter" demand curve has the higher elasticity. (Elasticity is ***not*** the same thing as slope although the two ideas are related.)

[9] Can you explain why this demand curve could not be vertical over all prices?

6. TOTAL REVENUE AND PED

1. Total revenue (TR) is price times quantity sold (P x Q) and as we move along a demand curve P and Q change, and so TR also changes. Firms and governments are interested in how TR is related to PED. Firms want to know what happens to sales if they change price. Governments want to know what happens to tax revenues if they impose a given sales tax on different commodities. The answers to these questions depend on the PEDs of the demand curves for the commodities in question. Table 2 shows what happens to TR when price changes for demand curves with different elasticities (we are not really interested in the "unit elastic" case, it dimply marks the difference between elastic and inelastic demand curves).

TABLE 2

Price Change	Inelastic Demand $0 < PED < 1$	Unit Elastic Demand $PED = 1$	Elastic Demand $PED > 1$
INCREASE	TR⇑	TR is CONSTANT	TR ⇓
DECREASE	TR⇓	TR is CONSTANT	TR⇑

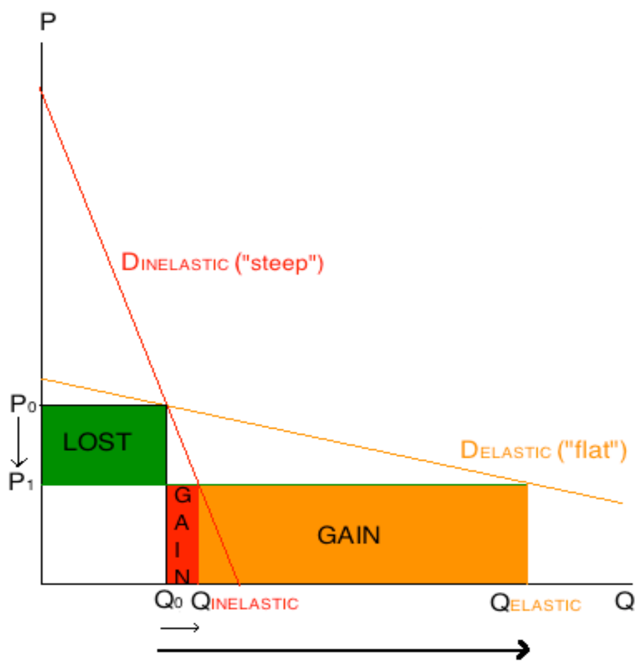

Figure 3

2. In Figure 3 above when the price falls from P_0 to P_1 firms lose revenue on existing sales (Q_0) because they only get P_1 for selling those units, whereas

before they got P_0 for each of those units, and $P_1 < P_0$. The green rectangle shows the lost TR in both cases (the same price change and the same quantity sold). But, the fall in price also increases TR, because more units are sold at the lower price. In the *inelastic* demand case because the quantity demanded is not very responsive to price changes, the sales increase is small and the increase in TR is also small, so small that the lost TR swamps it, and so TR falls (the green rectangle is larger than the thin red rectangle). In the *elastic* demand case, because the quantity demanded is very responsive to price changes, the sales increase is very large and the gain in TR is also very large, so large it swamps the lost TR, and TR increases (the green rectangle is smaller than the combined red and orange rectangles).

3. In the early years of the twentieth century Henry Ford revolutionized the infant automobile industry by switching to conveyor belt assembly lines in a giant plant. Because the conveyor belt system was a more efficient method of assembling automobiles Ford's unit cost fell. Economists call situations in which a large plant (large scale of production) yields lower unit costs than generating the same output with a number of smaller plants, "economies of scale". Although Ford's unit cost fell dramatically that would only lead to an increase in Ford's profits if he could sell the increased production. But that meant that Ford had to cut his model-T prices in order to move down the demand curve in order to increase sales sufficiently to cover the lower prices at which he was selling the model-T. But a fall in price will only lead to an increase in revenue if the demand for the product is elastic. In Ford's case he was successful and made a fortune. Table 3 shows model-T prices, sales, and revenues for 1909, 1914, and 1916. The PED of the Model-T was greater than one[10], the demand was elastic, and therefore the cut in price led to an increase in revenue. (In 2018 the Ford company is largely a manufacturer of trucks.)

TABLE 3

YEAR	PRICE ($)	SALES ('000)	REVENUE ($M)
1909	900	58	52.2
1914	440	472	207.8
1919	360	730	262.8

[10] If you calculate the PED it is only just over 2 (strictly -2). I don't think I have ever seen a PED greater than 20.

4. Ford also doubled the wage per hour that he was paying. This is an example of an "efficiency wage". The idea is that the firm pays higher wages and benefits and/or provides better working conditions. This increases the firm's labor costs. It also makes the firm's jobs more desirable compared to its competitors. That means that the firm will have an excess supply of workers and it can pick and choose amongst them to try and find the most productive ones. And supervision costs also drop. The workers have a strong incentive to work hard, because if they do not then when they are fired the next best job is much less attractive than the one that they just lost. (Note that this is the opposite of what happened in the draft: CM12.) Absenteeism will drop dramatically; by about 80% in Ford's case, because workers lose a lot if they are not at work, they even lose their jobs. "Turn-over" costs will drop dramatically because this job is the best one in town, and so the cost of hiring and training workers falls. (Again this is the opposite of what happens with a draft.) Firms like Microsoft are very anxious to make certain that their pay and benefits and work conditions are highly competitive with their rivals in the labor market for scarce IT workers.

7. DETERMINANTS OF PED

1. What determines price elasticity of demand? Substitution, substitution, substitution! (1) The more substitutes, and the better the substitutes, the more elastic the demand for X – it is easy to find an alternative to X when its price increases because there are many commodities that can be substituted in its place when it becomes relatively more expensive. In all my years teaching economics no one has ever been able to come up with a substitute for salt. Because salt has no substitutes it has a very low PED. Wool in the form of sweaters is a substitute for heating fuel and human fat in the form of weight limits on flight attendants, and paper and paint are substitutes for jet fuel. (2) The longer the period available to make an adjustment the greater the PED – it takes time to get rid of the "gas guzzler", or to replace the oil fired furnace. (3) The more that is spent on the commodity – the larger the share of total expenditure – the more elastic the demand (houses and cars versus salt, or peanuts, or ice cream cones). The larger the expenditure the more things the good or service has to compete with for our dollars – houses are, in some sense, substitutes for almost everything that we buy.

8. THREE MORE ELASTICITIES THAT YOU OUGHT TO KNOW

I) PES – the price elasticity of supply – is a measure of the responsiveness of the quantity supplied to the price of the good or service. PES is the percentage change in the quantity supplied divided by the percentage change in the price of X. PES is the supply side analog of the PED, but the PES is always positive, because the price of X and the quantity supplied of X are positively related. If $0 < PES < 1$ then supply is inelastic and the supply curve will (roughly) be "steep". If $1 < PES <$ "∞" then supply is elastic and the supply curve will (roughly) be "flat".

Supply is always more elastic in the long run than the short run; for example it takes time for oil to be discovered, wells to be sunk, oil to be drawn from the well, and then transported. When the first oil shock occurred in 1973-1974 the UK imported all of its oil and so did Nigeria, but in 2018 both countries exported oil. The supply effect only works if the price is allowed to adjust to the shortage. The gas price controls introduced in 1974 by President Nixon stopped the market working; gas had to be rationed by non-price means, usually by long waiting lines at gas pumps with high costs in terms of time wasted. The price controls removed the incentives to increase the quantity supplied in the short-run, and to increase the supply in the long run. On the demand side the failure to allow domestic gas prices to rise in the face of a large increase in world oil prices, stopped consumers economizing on the use of gas in the short run, and switching technologies in the long run.

II) IED – the income elasticity of demand – is a measure of the responsiveness of the quantity demanded to changes in income (strictly, real disposable income – correcting for inflation and after taxes). IED is the percentage change in the quantity demanded of X divided by the percentage change in income. Food has a relatively low income elasticity, but luxury cars and yachts have high income elasticities, we can manage with less expensive cars, but we must eat – although not in expensive restaurants. Undertaking and burial services are income inelastic. It is important for firms and for industries to know their IEDs because this will indicate how vulnerable they are to events like the Great recession.

III) CED – the cross elasticity of demand - measures the responsiveness of the quantity demanded of X to changes in **the prices of <u>other</u> goods and services, P_Y**: CED is the percentage change in the quantity demanded of X divided by the percentage change in the price of <u>Y</u> (a substitute or complementary commodity). Substitutes have positive CEDs (Coke and Pepsi) and complements have negative CEDs (gas and cars).

9. ESTIMATED PEDs

Various dates (1980s) and sources:

ELASTIC		INELASTIC	
GOOD/SERVICE	PED	GOOD/SERVICE	PED
Furniture	1.26	Utilities	0.92
Motor Vehicles	1.14	Drinks (All)	0.78
Professional services	1.09	Clothing	0.64
Transportation services	1.03	Tobacco	0.61
		Banking & Insurance	0.56
		Books etc.	0.34
		Food	0.12
		Oil	0.05

10. HOW TO CALCULATE PED

I will not ask you questions like this on the exam, because you probably would need more than 70 seconds to do the calculations, and many of you would need to use a calculator. This is a straightforward calculation so long as you remember the sign convention that economists use at this level of analysis. When the price changes are we going to calculate the percentage change in price in terms of the original price or of the new price? The answer is that we adopt a *convention*, which is to *use the average of the two prices and the two quantities*.

Example 1. The price of X is $10 and the quantity demanded is 200 tons. The price then drops to $5 and the quantity demanded increases to 2000 tons. The change in the price is -$5 ($\Delta P$ = -$5) and the change in the quantity demanded is 1800 tons (ΔQ = 1800 tons). But we will get different elasticities depending on whether we use P_0 ($10) and Q_0 (200) or P_1 ($5) and Q_1 (1800). The problem is that economists are usually thinking in terms of non-linear (curved) demand and supply curves and do the analysis using calculus so that they can calculate the PED at a specific point on the demand curve. But if we stay at P_0 and Q_0 then there is no change in P and Q. Therefore we will calculate the PED at the mid-point of the interval, half way between the two points: we will use the average of the two prices (and the average of the two quantities).

$PED_{av} = \%\Delta Q^d / \%\Delta P$

$= (\Delta Q/Q_{AV})\text{x}100 \div (\Delta P/P_{AV})\text{x}100$

$= [1800 \text{ tons} \div \{(2000 \text{ tons}+200 \text{ tons})/2\}] \text{ x}100 \div [[-\$5 \div \{(\$10+\$5)/2\}]\text{x}100]$

$= (1800 \text{ tons} /1100 \text{ tons}) \text{ x}100 \div (-\$5/\$7.5) \text{ x}100$

(Note that the tons will cancel and the dollars will also cancel and so we will end up with a pure number.)

= 163.6 ÷ - 66.7 = - 2.453 which we will interpret as 2.45 (we round down and we *ignore the minus sign*!). Because 2.45 > 1 demand is elastic.

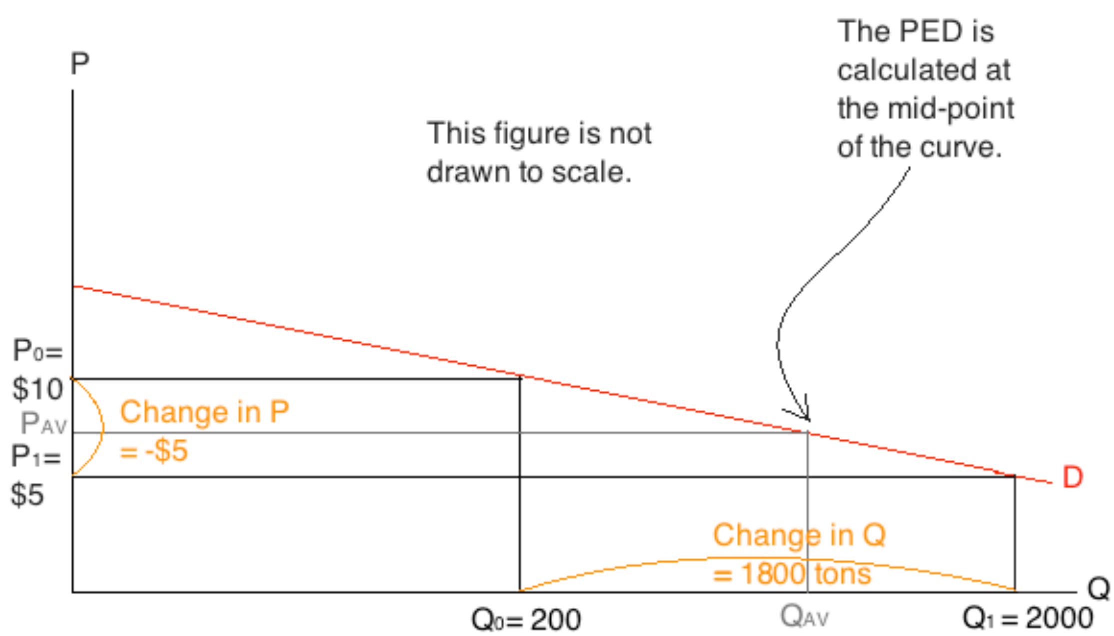

Figure 4

Example 2. The price of Y is $10 and the quantity demanded is 100 tons. The price of Y then drops to $5 and the quantity demanded increases to 110 tons. The change in the price is -$5 ($\Delta P$ = -$5) and the change in the quantity demanded is 10 tons (ΔQ = 10 tons). We will calculate the PED of Y at the mid-point of the interval, half way between the two prices and quantities: we will use the average of the two prices and the average of the two quantities.

PED_{av} = average%ΔQ^d/average%ΔP

= $(\Delta Q/Q_{AV})$x100 ÷ $(\Delta P/P_{AV})$x100

= [10 tons ÷{(110 tons+100 tons)/2}] x100 ÷ [[-$5 ÷ {($10+$5)/2}]x100]

= (10 tons /105 tons) x100 ÷ (-$5/$7.5) x100

= 9.524 ÷ - 66.7 = - 0.1428 which we will interpret as 0.14 (we round down and we *ignore the minus sign*!). Because 0.14 < 1 demand is (very) inelastic. (3,926)

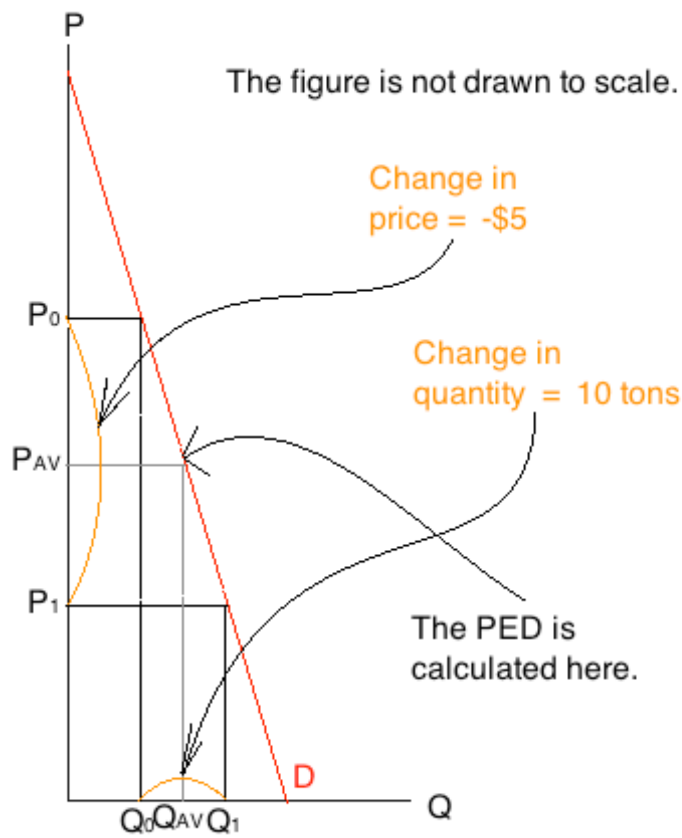

Figure 5

http://money.cnn.com/2013/01/10/news/companies/cigarette-tax-new-york/

http://www.theguardian.com/world/2014/feb/14/nepal-slashes-cost-climbing-mount-everest http://www.nytimes.com/2005/10/13/business/13scene.html?_r=0

http://www.cato.org/blog/laffer-curve-strikes-again