A 11. SINGLE VARIABLE CALCULUS ASSIGNMENT

FOLLOW THE STANDARD INSTRUCTIONS FOR DOING THE ASSIGNMENTS.

1. Explain carefully, using a diagram, the meaning of the term the derivative of the function f at a point in its domain, provide a formal definition of such a derivative, give two intuitive interpretations of the information conveyed by derivatives, and give two examples of derivatives which arise in economics (one from microeconomics and one from macroeconomics).

2. What is likely to be the most serious limitation of the differential calculus in economic contexts? Why, nonetheless, do economists rely so much on differential calculus when doing economics – in both theory and in applied contexts?

3. Using the standard supply and demand model calculate dQ/da, dQ/db, dQ/dc, $dQ/d\delta$, dP/da, dP/db, dP/dc, and $dP/d\delta$ (where δ is the slope coefficient of the supply curve).

4. Starting from the definition of equilibrium income in *the open economy income expenditure model with a marginal tax rate,* use calculus to determine the:

   (a) autonomous government expenditure,
   (b) autonomous tax,

   (c) balanced budget,

   (d) marginal tax,

(e) and marginal import multipliers.

Use simple *algebraic* arguments to sign the multipliers, to place limits on their numerical magnitudes (e.g. are they less than one?), and to show the relationships between the multipliers (e.g. is one multiplier larger than another?).

A 12 HIGHER ORDER DERIVATIVES AND EXTREMIZATION: SINGLE VARIABLE CALCULUS ASSIGNMENT

FOLLOW THE STANDARD INSTRUCTIONS FOR DOING THE ASSIGNMENTS.

1. Draw a *sketch* of the graphs following functions:

a) $f: R \to R$

s.t. $f(0) = 0$, $f'(x) > 0$, $f''(x) > 0$

b) $f: R \to R$

s.t. $f(0) = 0$, $f'(x) > 0$, $f''(x) = 0$

c) $f: R \to R$

s.t. $f(0) = 0$, $f'(x) > 0$, $f''(x) < 0$

d) $f: R \to R$

s.t. $f(0) = 0$, $f'(x) < 0$, $f''(x) < 0$

e) $f: R \to R$

s.t. $f(0) > 0$, $f'(x) > 0$, $f''(x) = 0$

f) $f: R \to R$

s.t. $f(0) > 0, f'(x) > 0, f''(x) < 0$

g) $f: R \rightarrow R$

s.t. $f(0) < 0, f'(x) < 0, f''(x) < 0.$

2. Give a verbal description of, and draw a sketch of, the graph of the following function:

$$y = f(x), \quad f' > 0, \quad f'' < 0, \quad f(0) = 0 \quad (x \in R^0).$$

3. Prove that at the maximum point on the AP_L curve for labor, $MP_L = AP_L$. (Hints: What do you know about the slope of the AP_L curve at its maximum point? If you are trying to prove something about a slope what mathematical operation must you perform? Remember that Total Product of labor = TP_L = Q = the output of the firm. Also remember that you may not divide by zero and so $Q > 0$.)

4. What can we deduce if we know that a firm is operating at its profit maximizing level of output?

5. Show that the profit maximization condition for the output side of the market is equivalent to the (short run, one variable input) profit maximization condition for the firm's input hiring decision.

A 13. ELASTICITIES ASSIGNMENT

FOLLOW THE STANDARD INSTRUCTIONS FOR DOING THE ASSIGNMENTS.

1. Let f: $R^0 \rightarrow R^0$

where $Q^d = f(P) = a + bP \quad a > 0, b < 0$.

What are:

(a) f^{-1}

(b) 1/f

(c) $TR = TR(Q) = P.Q \quad (Q \in R^0)$?

(Hint: re-read MR2 on Functions.)

2. Let

$$f: R^0 \rightarrow R^0$$

$$\text{s.t. } Q^d = f(P) \quad f'(P) < 0$$

be the demand for widgets function. Determine dTR/dP and show how it is related to the PED for widgets. Why is this derivative likely to be of interest to firms producing widgets?

3. Show that

$$MR = \frac{dP}{dQ} . Q + P.$$

Draw a diagram with a negatively sloped inverse demand curve. Choose two prices, $P_1 < P_0$ and determine the corresponding total revenues at these prices graphically. Show how the total revenue areas change as a consequence of the price change. Use your diagram to *explain* the formula for MR given in the first sentence.

4. The demand for wedgets is given by

$$f:\ R^0 \dashrightarrow R^0$$

$$\text{s.t. } P^d = f(Q) = a + bQ \quad a,b \in R, \quad a > 0,\ b < 0.$$

Calculate the MR function and show that it bisects the horizontal axis between the origin and $-a/b$. Show that the PED for wedgets is *not* constant and is *not* equal to b. Explain why this is the case.

5. Prove that PED is *not* constant for *any* negatively sloped demand curve.

6. If the demand for □idgets is given by

$$f:\ R^+ \dashrightarrow R^+$$

$$\text{s.t. } Q^d = f(P) = aP^b \quad a, b \in R, \quad a > 0,\ b < 0$$

determine the PED of □idgets. What can you say about PED in this case? Draw a *sketch* of the corresponding demand curve in the case where a = 1, b = -1.

7. Show that a profit maximizing non-competitive firm will not produce where the price elasticity of demand is greater than

minus one. (Hint: TC is a monotonically increasing function, i.e., TC' > 0 ($Q \in R^0$). What does this imply about MC? Use the relationship between MR and PED.)

8. The three supply curves illustrated in the diagram below are **all** unit elastic. Show that this is true and show that if the vertical intercept of supply curve is above the horizontal axis **PES<1** (i.e. supply is *inelastic*), while if the vertical intercept lies below the horizontal axis **PES>1** so that supply is *elastic*.

This diagram consists of three straight line supply curves, S, S', S'', one very flat and close to the horizontal (P) axis, one very steep and close to the vertical (Q^S) axis, and one drawn at 45 degrees to the horizontal axis. **All three supply curves pass through the origin**.

The diagram is missing. Ask me for this if I do not give it to you.

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9. Prove that a *revenue* maximizing firm will **only** produce where PED = -1. (Hints: What do you know about the slope of the TR curve at its maximum point? What is the slope of the TR curve called? Is there a connection between MR and PED?)

10. Derive a formula that relates price to MC and PED for a profit-maximizing firm.

A 14. SINGLE VARIABLE EXTREMIZATION ASSIGNMENT

FOLLOW THE STANDARD INSTRUCTIONS FOR DOING THE ASSIGNMENTS.

1. Let $y = f(x) = 3x^6$ $(x \in R)$. Calculate the derivatives of f of orders four through eight *if* they exist.

2. Locate any local extrema of:

   (a) $y = x^4 - 2x^2 + 2$ $(x \in R)$

   (b) $y = 3x^4 - 4x^3$ $(x \in R)$

3. Use algebra and calculus notation to represent the function whose graph is given below:

This graph will be added later. Ask me about it!

The diagram is missing. Ask me for it if I do not give it to you.

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4. For any real function f explain what $f(x)$, $f'(x)$, $f''(x)$,$f'''(x)$ tell us about the function (and its derived functions).

5. Describe the mathematical procedure known as the nth-derivative test.

6. Explain what the following terms mean:

   (a) A stationary point of a function.

   (b) A local maximum of a function.

   (c) A global minimum of a function.

   (d) An interior extremum of a function.

   (e) An end-point extremum of a function.

7. Comment on the following:

   (a) If $f'(x_0) = 0$ then f has an extremum at the stationary point x_0.

   (b) $f(x_0) > f(x)$ therefore $f(x_0)$ is a global maximum.

   (d) If $f'(x_0) = 0$ and $f''(x_0) = 0$ then we have located a horizontal point of inflection.

   (e) If f achieves a local minimum at x_0 then $f'(x_0) = 0$, and $f''(x) > 0$.

8. Prove that at the *minimum point* on the **AVC** curve **MC =AVC**. (Hints: What do you know about the slope of the AVC curve at its maximum point? If you are trying to prove something about a slope what mathematical operation must you do?)

9. Prove that at the *maximum* point on the **AR** curve, **AR = MR**.

10. What can you deduce if you know that a profit maximizing monopolist is in equilibrium? Would your answer be different for a perfectly competitive firm?

11. Show that a **revenue** maximizing firm will produce at the output where PED = -1.

12. Show that the supply curve of a perfectly competitive firm is equal to that part of its MC curve that lies above the minimum point of the AVC curve in the short-run, and above the minimum point of the AC curve in the long-run.

13. Show that the profit maximization condition for the **output** side of the market is equivalent to the short run, one variable input, profit maximization condition for the firm's **input** hiring decision.

A 15. FUNCTIONS OF TWO VARIABLES ASSIGNMENT

FOLLOW THE STANDARD INSTRUCTIONS FOR DOING THE ASSIGNMENTS.

1. Explain carefully what it means to say:

   (a) That w is a function of two variables u and v; and
   (b) That y is a function of n variables $x_1, x_2, \ldots x_n$.

2. (a) Define the first-order partial derivatives of

   $$z = f(x,y) \quad (x,y) \in R^2$$

   (b) What, intuitively, do the first-order partial derivatives of a function of two variables tell us about the function?

   (c) Give two examples of first-order partial derivatives that arise in microeconomics.

3. If $C = C(Y,R,W)$ where $C(0,0,0) = 0$ and $C_Y > 0$, $C_{YY} < 0$, $C_R < 0$, $C_W > 0$ $C,Y,R,W \in R^0$ and where C is the level of consumption, Y is the level of (real, disposable) income, R is the real rate of interest, and W is the stock of (real, private) wealth, then *describe* what the consumption surface looks like.

4. Calculate **all** of the first order partial derivatives of the following function (assume that all variables are real numbers):

   $$z = 2x^2 + 3xy + 2y^3$$

5. Calculate all first and second order partials of:

$$Q = f(L,K) = 2L^3 + 4L^2K - LK^3 + 3K^4$$

6. A firm has a production function of the form:

$$Q = 10L^{2/3}\,K^{1/3} \quad L,K > 0 \;\; (L,K \in R^2)$$

(a) Calculate the marginal product functions, their slopes, and their curvatures.

(b) What are the marginal products when 2 units of labor are employed with 5 units of capital?

7. Calculate all first, second, and third order partials of:

$$Q = f(L,K) = AL^{\alpha}K^{\beta} \quad A,\alpha,\beta>0,\; 0<\alpha,\beta<1 \;\; A,\alpha,\beta \in R^0.$$

What interpretations can be given to these partial derivatives?

8. Calculate all first and second order partials of:

$$U = f(x,y) = 2x^{1/2}y^{1/2} \quad (x,y) \in R^2 \;\; f(x,y)=0.$$

9. Calculate all first, second, and third order partials of:

$$U = U(x,y) = Ax^{\alpha}y^{\beta} \quad A,\alpha,\beta>0,\; 0<\alpha,\beta<1 \;\; A,\alpha,\beta \in R^0,$$

What interpretations can be given to these partial derivatives?

10. What is a "level curve" of a function of two variables? How can you use level curves to illustrate the behavior of the surface that is the graph of the function?

11. Explain, using the concept of a function of n-variables, what $\partial Q^d/\partial P$ and $\partial Q^d/\partial Y$ tell us about a demand function (where Y represents real income and Q^d and P have their usual economic interpretation).

12. Explain how a partial derivative is related to a "trace" of a function. (A "trace" is a plane parallel to one of the axes.)

13. Assume that $Q^d{}_X = f(P_X, P_S, P_C, Y)$ is a well behaved demand function. Explain what will happen to the two-dimensional *trace* relating $Q^d{}_X$ to Y (nominal income):

    (a) when Y alone changes, and

    (c) when P_X alone changes.

A 16. TWO VARIABLE EXTREMA ASSIGNMENT

FOLLOW THE STANDARD INSTRUCTIONS FOR DOING THE ASSIGNMENTS.

Find the (x,y) coordinates which correspond to stationary points for the following functions of two non-negative real variables x and y and determine what type of stationary points you have located.

1. (a) $z = f(x,y) = -x^2 + 10x + 2xy - 8y - 2y^2 + 5$

   (b) $z = f(x,y) = 2x^2 - 16x + 2xy - 20y + y^2 - 8$

   (c) $z = f(x,y) = x^2 - 6x + 8y + y^2 + 3$

2. A monopolist has two plants, A and B. Use calculus to determine how to allocate the firm's profit maximizing output between the two plants.

3. A monopolist can sell its output in two separated markets with different demand curves. Use calculus to determine how the optimal prices in the two markets depend on their respective elasticities of demand.

A 17. TWO VARIABLE CONSTRAINED EXTREMA ASSIGNMENT

FOLLOW THE STANDARD INSTRUCTIONS FOR DOING THE ASSIGNMENTS.

1. Show that a consumer's constrained equilibrium occurs where the slope of the budget constraint is equal to the slope of the highest attainable indifference curve.

2. Show how a firm that maximizes output subject to a cost constraint will choose its optimal input mix.

3. Show that if the firm minimizes cost subject to an output constraint (i.e. it is restricted to the highest isoquant achieved in the previous question) that it will hire the same combination of inputs as in the previous question.