ECONOMIC THEORY 11

ELASTICITY

INTRODUCTION

1. Economists want to answer questions such as: how much does Y change when X changes? That is, we would like to be able to say something about the size of ΔY brought about by a change in ΔX. This would seem to take us out of the qualitative world in which we have been operating until now and into the world of quantitative analysis. However, as we shall see, there are qualitative things that can be said about this topic.

2. At first sight it would appear that we could use the slope of the graph of the function relating Y to X to answer our question since the slope coefficient is $\Delta Y/\Delta X$, but unfortunately **the slope coefficient is not a pure number because its magnitude depends on the units in which Y and X are measured.** (Since you can now do simple differential calculus we will use our derivative notation to indicate the slope of the graph of the function at a point on that graph, i.e. we will use dY/dX rather than $\Delta Y/\Delta X$, although for *linear* functions the two ways of measuring the slope yield the same answer.)

   This point about units of measurement may be more obvious if we ask a slightly different question: which is more responsive to changes in price – the quantity demanded of Cadillac Sevilles or the quantity demanded of table salt? Specifically, say that we know that the price of Cadillac Sevilles fell by $3,000 last year and that sales increased by

10,000 cars, whereas the price of table salt fell by 10 cents and sales increased by 2 million pounds. Obviously a comparison of these changes is not possible because cars and pounds of salt are incommensurable.

3. The problem is much more pervasive than the last example suggests because the units that we use to measure the Y and X variables determine the size of the slope coefficient of the demand function. Consider the demand function for some mineral such as copper,

$$f: R^0 \to R^0$$

$$\text{s.t. } Q_X^D = f(P) = 100 - 5P$$

where Q is measured in US short tons (I S.T. = 2,000 lbs.) per annum and P is measured in thousands of US dollars per short ton ($000/S.T.) Then the **slope coefficient**, $dQ_X^D/dP = -5$, tells us that an increase in the price of copper of $1,000 (per S.T. per year) will cause the quantity demanded of copper to fall by 5 short tons per year, i.e. by 10,000 pounds per year. The vertical intercept term tells us that even if the price of copper were to fall to zero consumers would still not consume more than 100 S.T.s of copper per year. We can also see that if the price of copper per S.T. were to rise to 20 (thousand dollars per S.T. per year) then demand would be completely choked off and consumers would stop buying copper, i.e. $Q_X^D(20) = 0$.

Now say that we change our units of measurement of copper from short tons per year to pounds per year. Then we have to rewrite **the equation** of the demand function or we will end up with a different demand function. The new demand equation must still show consumers wanting to consume 100 S.T.s =

200,000 pounds of copper per year if the price of copper falls to zero. Further the new demand equation must show that the quantity of copper demanded must fall by 5 S.T.s = 10,000 pounds per year if the price of copper increases by $1,000 per S.T. (And the equation must tell us that the quantity demanded of copper will fall to zero if the price of copper rises to 20 (= $20,000) per S.T. (= 2,000 lbs.)) The demand function for copper now takes the form:

$$f: R^0 \rightarrow R^0$$

$$\text{s.t. } Q_X^D = f(P) = 200{,}000 - 10{,}000P$$

and the new slope coefficient becomes $dQ_X^D/dP = -10{,}000$ pounds (= 5 S.T.s) per thousand US dollars annum. Obviously the price responsiveness of the demand for copper has not changed because of the change in the units in which the demand for copper is measured but the slope coefficient has changed by a factor of 2,000.

Alternatively we could revert to measuring the consumption of copper in short tons but measure the price in dollars rather than thousands of dollars. Again we must come up with a new demand equation. The new demand function takes the form:

$$f: R^0 \rightarrow R^0$$

$$\text{s.t. } Q_X^D = f(P) = 100 - 0.005P.$$

The new equation tells us that if the price of copper falls to zero then consumers will wish to consume 100 S.T.s of copper per year. We can also see that if the price of copper were to increase by $1 per short ton then the quantity demanded would fall by 0.005 S.T.s = 10 pounds whereas if the price of copper were to increase by $1,000 then the quantity demanded would fall by 5 S.T.s. While the slope coefficient has changed radically relative to the two previous examples we can see that the responsiveness of the demand for copper to price changes has not altered.

The three slope parameters -- -5, -10,000, and - 0.005 – are numerically quite different but they all measure the same degree of price responsiveness. It should be clear that we can not easily use the slope coefficient as a measure of how responsive the dependent variable is to a change in an independent variable.

4. You will remember from ECON 206 that economists use the concept of **elasticity** to **measure the responsiveness of a dependent variable to a change in an independent variable**. (The idea can be found in Augustin Cournot's superb "Researches into the Mathematical Theory of Wealth" (1838) although the terminology seems to have originated with Marshall's "Principles" (1890) Ch.4 "The Elasticity of Wants".) Note the complete generality of the definition – we can have price elasticities of demand and supply, interest elasticities of investment and saving, and wage elasticities of the demand and supply of labor, etc. What Cournot and Marshall realized was that we can measure and compare the responsiveness of Y to X or of Z to W **so long as we calculate all of the changes in *percentage* terms**.

5. The general definition of **the elasticity of Y with respect to X** is **the percentage change in Y brought about by a one percent change in X**. In symbols we have:

$$E_{YX} = \%\Delta Y / \%\Delta X$$

$$= (\Delta Y/Y).100/(\Delta X/X).100$$

$$= (\Delta Y/Y)/(\Delta X/X)$$

$$= (\Delta Y/Y).(X/\Delta X)$$

$$= (\Delta Y/\Delta X).(X/Y).$$

Taking the limit of $\Delta Y/\Delta X$ as ΔX approaches zero allows us to calculate a **point elasticity**, the elasticity of Y with respect to X at a point on the graph of the function that relates Y to X:

$$\mathbf{E_{YX} = (dY/dX).\ (X/Y)}$$

This is the elasticity formula that we will use throughout ECON 208.

6. In this course we will be primarily concerned with *the price elasticity of demand*, **PED**. However you should also know how to define the *income elasticity of demand*, **IED** which is the percentage change in the quantity demanded with respect to a one per cent change in income. Unlike PED it is the **sign, not the magnitude,** of IED that is important. If **IED > 0** then the increase in income causes the quantity demanded to increase and therefore the good or service in question is **normal**, whereas if **IED < 0** then the good or service is **inferior** because the increase in income causes the quantity demanded to fall. In ECON 306 you will learn about Engel curves (not Marx's collaborator) and you may draw analogies between PED and the demand curve and IED and the Engel curve – you may even prove some

theorems about IED. But this is all we have to say about IED in ECON 208.

Cross elasticity of demand, **CED_{YX}**, is the percentage change in the quantity demanded of **Y** that is brought about by a one per cent change in the price of **X**. Again it is the **sign**, **not the magnitude**, of CED that is important. If **CED > 0** then an increase in the price of X causes an increase in the demand for Y and so X and Y are **substitutes**, whereas if **CED < 0** then X and Y are **complements**.

Price elasticity of supply, **PES**, is the percentage change in the quantity supplied brought about by a one per cent change in the price of the good or service.

PRICE ELASTICITY of DEMAND

1. Price elasticity of demand, PED, is a measure of the responsiveness of the quantity supplied to changes in the price of the good. It is defined as the percentage change in the quantity demanded brought about by a one per cent change in the price of the good or service. In symbols we have:

$$\textbf{PED} = (\Delta Q_X{}^D/\Delta P_X).(P_X/Q_X)$$

$$= \boldsymbol{(dQ_X{}^D/dP_X).(P_X/Q_X)}$$

in the limit when the change in P_X is vanishingly small. The latter expression is what we call the point PED since it refers to the point (Q,P) on the demand curve. (In ECON 206 you also learned about arc elasticity of demand, AED, for which you can refer to the Henry Ford handout. You will not be asked about AED on the exams.)

2. PED is a **non-positive** real number so long as the good in question is not a Giffen good, because non-Giffen goods have **negatively** sloped demand curves and so $dQ_X^D/dP_X \leq 0$. (P and Q are both non-negative real numbers.) A perfectly inelastic demand curve has a PED of zero. If PED lies **between zero and -1** then demand is **in**elastic, and if PED is **less than -1** then the demand is **elastic**. (See Figure 1.) If PED = -1 demand is **unit** elastic. If $-1 < \text{PED} \leq 0$ then demand is **inelastic**. If $-\infty < \text{PED} < -1$ then demand is **elastic**. Be careful when writing down these equalities because our intuitions (including mine!) do not work well with negative numbers.

3. PED is related to total revenue (TR) as the Henry Ford handout shows. If we **lower price** and **demand is elastic** then **TR rises** because the gain in revenue from the higher sales volume ($Q_X^D > 0$) more than compensates us for the lost revenue caused by selling the original units at the lower price. (We are assuming that all units of the good sell for the same price, the price of the last (marginal) unit sold). You should be able to show that **TR falls** if we **cut the price** and **demand is inelastic** and that **TR is constant** if **demand is unit elastic**.

4. Say we have a standard differentiable demand function:

$$f: R^0 \rightarrow R^0$$

$$\text{s.t. } Q_X^D = f(P)\ f'(P) < 0.$$

Then it is clear that PED is related to, but not the same as, the slope of the demand curve, i.e. $\text{PED} \neq dQ_X^D/dP_X$, because PED includes the term P/Q. It should also be clear that PED is, in general, not constant **even if the demand curve is linear and has a constant slope**, because *the P/Q term changes value as we move along the demand curve*. If we specify a linear demand function:

$$f: R^0 \rightarrow R^0$$

$$\text{s.t. } Q_X^D = f(P) = a + bP \quad a > 0, b < 0$$

Then clearly the slope of the corresponding demand curve is $dQ_X^D/dP = b < 0$ where b is a real **constant** but PED = **b** (P/Q) which varies as P/Q varies. (You should be able to show that PED approaches $-\infty$ as Q approaches zero (as we get closer and closer to the P axis) and that PED = 0 when P = 0 on the Q axis. (See Figure 1.)

PRICE ELASTICITY of SUPPLY

1. Price elasticity of supply is defined to be the percentage change in the quantity supplied brought about by a one per cent change in the price of the good or service. Since supply curves are (usually) positively sloped PES ≥ 0. If PES = 0 then supply is completely inelastic, only one quantity will be supplied irrespective of the price and the supply curve is **horizontal** (using the correct mathematical coordinates). If PES = ∞ then supply is "infinitely" elastic and the supply curve is **vertical** (there is only one price at which this commodity will be supplied). If PES = 1 then supply is unit elastic and passes through the origin if the supply curve is linear. If $0 <$ PES < 1 then supply is inelastic -- quantity supplied is less than proportionally responsive to changes in price. If $1 <$ PES $< \infty$ then supply is inelastic -- quantity

supplied is more than proportionally responsive to changes in price.

Figure 1

$-\infty$ ____________________________ -1 ____________________________ 0

$-\infty < PED < -1$ ELASTIC $PED = -1$ UNIT $-1 < PED < 0$

INELASTIC

$\infty > |PED| > 1$ (ELASTIC)

$|PED| = 1$ (UNIT ELASTICITY)

$1 > |PED| > 0$ (INELASTIC)

Where $|PED|$ is the **absolute** or **numerical** value of the PED.