ECONOMIC THEORY 10

MODEL 3

An OPEN ECONOMY with a MARGINAL TAX RATE

1. Model 3 incorporates a simple foreign trade sector into Model 2. Model 3 is **the** ECON 208 macroeconomic model and you should **always** use it when asked to construct a macro/income-expenditure model in this class -- unless I tell you otherwise. Models 1 and 2 were preliminary versions of Model 3 that, I hope, will make the transition to the final version easier, i.e. Models 1 and 2 were just special cases of Model 3. When working with this model you need to continually remind yourselves that, like its predecessors, it consists of nothing more than two linear functions and an equilibrium condition. Therefore you know how it works mathematically. Model 3 is our last -- although still terribly inadequate -- attempt to capture how the US economy works.

2. Model 3 adds exports and imports to Model 2 but we are still modeling a barter system and so there are no financial assets, no capital flows, and no exchange rate.

3. We will continue to use Model 2's tax function and so there is a **marginal tax rate**.

4. Model 3 adds a new sector to our model -- the **foreign trade sector**. Domestic firms can now sell part of their output to foreigners -- so we add an **export function** to model the foreigners' demand for our goods. And our households, firms, and government agencies can now purchase goods

from foreign firms and so we need to incorporate an **import function** into our model. Nothing else in the model changes -- we have the same AS function, the same equilibrium condition as in Models 1 and 2, etc.

5. Our **export function** is very primitive, because we want to keep the model as simple as possible. Exports obviously depend upon the incomes of foreigners, the relationship between our prices and foreign prices, the exchange rate, and other factors too. Our preoccupation with simplicity means that we will have to take all of these factors as given exogenously and so the export function is just a constant function (see Figure 1):

$$X = X(Y) = X_0 \qquad X_0 > 0.$$

Exports are now added to the other components of autonomous AD and are shown entering the firm sector from outside the circular flow diagram in Figure 2.

6. **Imports** are a *leakage* from the circular flow. We will assume that imports are a linear function of **disposable** income. We do this because we assume that the bulk of imported goods go to the household sector and that imports by the firm and government sectors are dominated by imports that are substitutes for domestically produced consumer goods. Alternatively we could make imports depend on Y, but doing so would not materially alter the predictions of our model. The **import function** has the standard linear algebraic form:

$$M = M(Y) = M(Y^D) = M_0 + mY^D \qquad M_0 < 0 \qquad 0 < m < 1.$$

Given that $Y^D = - T_0 + (1\text{-}t)Y \quad T_0 > 0$ and $0 < t < 1$ we can write the import function as:

$$M = M_0 + m(-T_0 + (1-t)Y)$$

$$M = M_0 - mT_0 + m(1-t)Y \qquad M_0 - mT_0 > 0 \qquad 0 < m(1-t) < 1.$$

$M_0 - mT_0$ is autonomous imports (M^{AUT}) -- the **vertical intercept** of the import function -- and $m(1-t)Y = M^{IND}$ is induced imports (see Figure 3). We can again incorporate a *stylized fact* into our analysis. We will assume that the *marginal propensity to import out of disposable income* (**m**) is smaller than the marginal tax rate (t), which in turn is smaller than the marginal propensity to consume out of disposable income (c): $0 < m < t < c < 1$. Therefore we can deduce that $m(1-t) < c(1-t)$.

Imports are a leakage from the circular flow and so we show them flowing out of the household sector box in Figure 3 (induced imports, M^{IND}) and being subtracted from autonomous AD (autonomous imports $M_0 - mT_0$).

7. Model 3's AD function incorporates foreign trade. The new **AD function** is more complex than the AD function of Model 2 because it allows for exports and imports but if you build it up from first principles it should ultimately make sense to you. AD consists of the demand for the output of domestic producers from households, firms, the government sector, and foreigners, less that part of domestic expenditures that goes to purchase goods from foreign firms. So the aggregate demand function, AD, takes the form:

$$AD = f(Y) = C(Y) + I(Y) + G(Y) + X(Y) - M(Y)$$

$$= C_0 - cT_0 + c(1-t)Y + I_0 + G_0 + X_0 - (M_0 - mT_0 +$$

$m(1\text{-}t)Y)$

$$= C_0 + I_0 + G_0 + NX_0 - cT_0 + mT_0 + c(1\text{-}t)Y - m(1\text{-}t)Y$$

$$= A_0 + G_0 + NX_0 - (cT_0 - mT_0) + c(1\text{-}t)Y - m(1\text{-}t)Y$$

$$= A_0 + G_0 + NX_0 + (m - c)T_0 + [c(1\text{-}t) - m(1\text{-}t)]Y$$

$$AD = AD^{AUT} + AD^{IND}$$

where $AD^{AUT} = A_0 + G_0 + NX_0 + (m - c)T_0 > 0$ is the **vertical intercept** ($VIAD^m$) of the new aggregate demand curve, AD^m, and where $A_0 = C_0 + I_0$, $NX_0 = X_0 - M_0$ = **net autonomous exports**, and $0 < c(1\text{-}t) - m(1\text{-}t) < 1$ is the **slope coefficient** of the AD^m curve, and where $AD^{IND} = [c(1\text{-}t) - m(1\text{-}t)]Y$ is induced aggregate demand. Note that we write $-(c\text{-}m)$ as $(m\text{-}c < 0)$. The new aggregate demand curve is plotted in Figure 4 where we have *assumed* that the intercepts of the three aggregate demand curves coincide. (It is difficult to say, a priori, whether the vertical intercept of the new AD curve will lie above or below the vertical intercepts of AD and AD^t).

8. The rest of Model 3 is essentially the same as Models 1 and 2 so I leave its full development as series of exercises (see the Model 3 section of the Assignment 8 Key).

9. The first part of this section of the key sets out **Model 3 in algebraic terms**.
10. The third part of this section of the Key provides the formula for the **equilibrium level of income** for Model 3:

$$Y^e = \frac{1}{1\text{-}c(1\text{-}t) + m(1\text{-}t)} (A_0 + G_0 + NX_0 - (c\text{-}m)T_0)$$

or

$$Y^e = \frac{VIAD^m - VIAS^m}{SCAS^m - SCAD^m}$$

where $VIAD^m = A_0 + G_0 + NX_0 - (c\text{-}m)T_0$, $VIAS^m = 0$, $SCAS^m = 1$, and $SCAD^m = c(1\text{-}t) - m(1\text{-}t)$. (Note the sign changes between the denominator of the Y^e equation and the $SCAD^m$ equation. You should be able to explain why these signs are different.)

11. The next section of the last part of the Key derives the **QCS multipliers for Model 3**. These multipliers are derived from the **basic QCS equation for this model**:

$$\Delta Y^e = \frac{1}{1\text{-}c(1\text{-}t) + m(1\text{-}t)} (\Delta A_0 + \Delta G_0 + \Delta NX_0 + (m\text{-}c)\, \Delta T_0)$$

In order to distinguish Model 3's multipliers from those of Model 1 we attach an *m* superscript to them. k_t^m and k_m are **pivot** multipliers, since changing the marginal tax rate or the marginal propensity to import, m, cause the AD^m curve to pivot about its vertical intercept.

An increase in t causes Y^e to fall as the aggregate demand curve pivots upwards so k_t^m is negative. The effect of the change in the marginal tax rate is dependent on the level of income at which the tax rate increase occurs (see Figure 6). That is, an increase in t from say, 0.25 to 0.50, will raise more tax if the initial income level is \$10t than if it is only \$5t. The larger the increase in tax revenues the greater the reduction in disposable income, induced consumption (induced AD) and the larger the resulting fall in the equilibrium level of income. So *k_t^m depends positively on the equilibrium level of income*.

Similarly *an increase in the marginal propensity to import* will reduce demand for domestically produced goods and so will lower the equilibrium level of income. Again the AD^m curve pivots downwards. Therefore the k_m multiplier is negative. We use the notation k_m rather than k_m^t since this multiplier **must** belong to Model 3. Note that the impact of a change in the marginal propensity to import depends positively on the size of disposable income. The reasoning is the same as we used in the previous paragraph (see Figure 6). Note that *m is not a policy parameter* -- it is a behavioral parameter like c. We have calculated k_m to illustrate how the calculation may be done.

12. The last section of the Key shows that as we move from Model 1 to Model 2 to Model 3 the economy becomes progressively more **damped**. This means that the multiplier effects of, for example, a change in autonomous government expenditures get progressively smaller as we move from Model 1 (k_G) to Model 2 (k_G^t) to Model 3 (k_G^m). However there is an exception to this result, $k_{BB}^m > k_{BB}^t$ (I do not have an intuitive economic explanation for this and would be very interested if you can come up with one.) The government expenditure multiplier for Model 3 is only about 2.1 if we use c=4/5, t=1/4, and m= 3/20 = 0.15. While this is a far cry from our initial multiplier of 5 it is still much larger than most estimates of the actual multiplier for the US which is almost certainly less than two and probably not much larger than one.

Figure 3

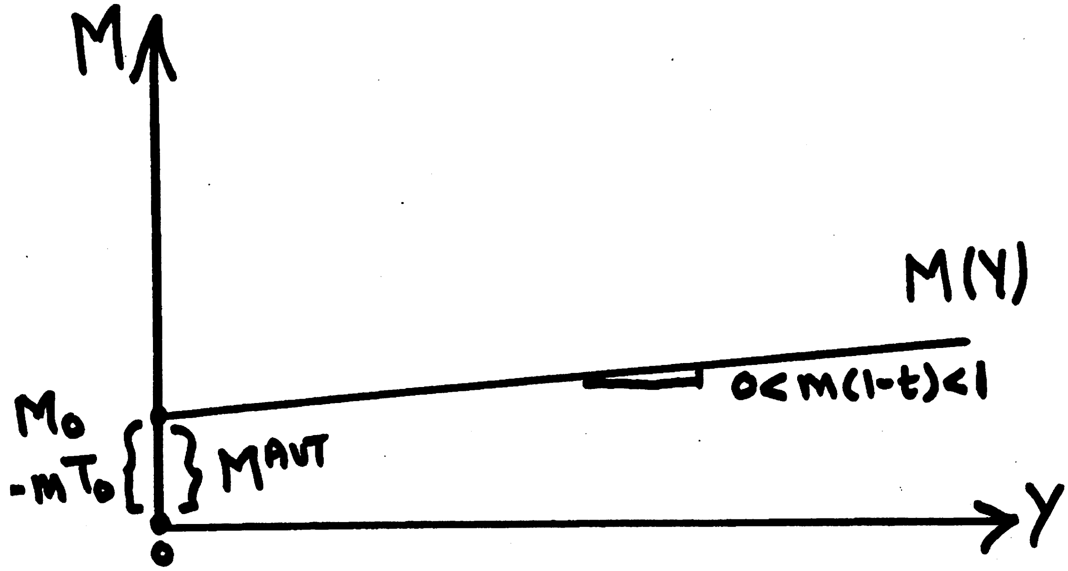

Figure 4

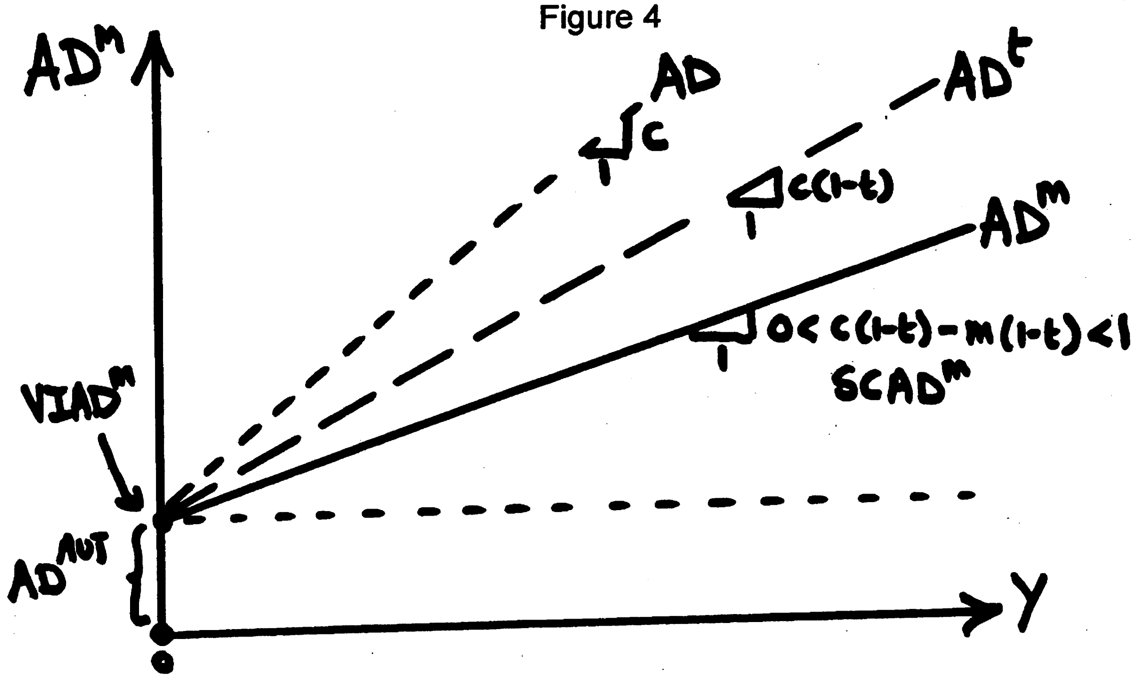

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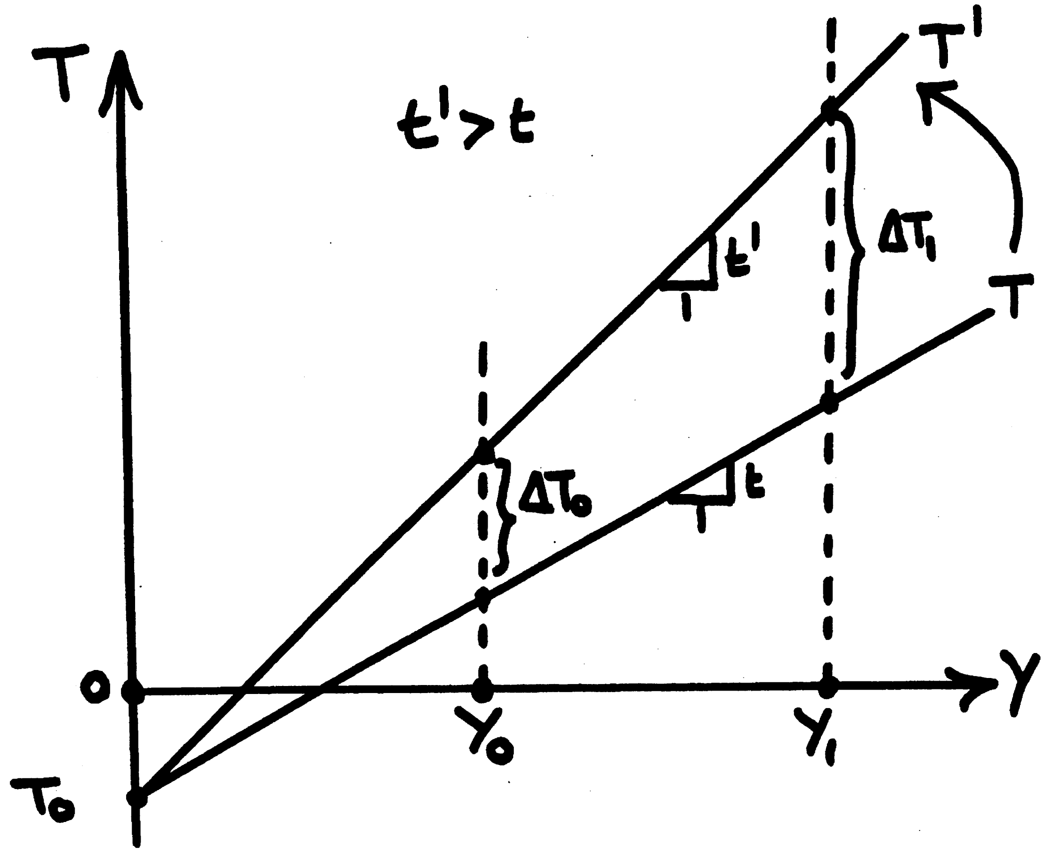
Figure 5



An *increase* in the marginal tax rate from t to t', i.e. $\Delta t = t' - t > 0$, will raise more tax revenue at a high income level, like Y_1, than at a low income level, like Y_0, i.e. for a given $\Delta t > 0$, $\Delta T_1 > \Delta T_0$.

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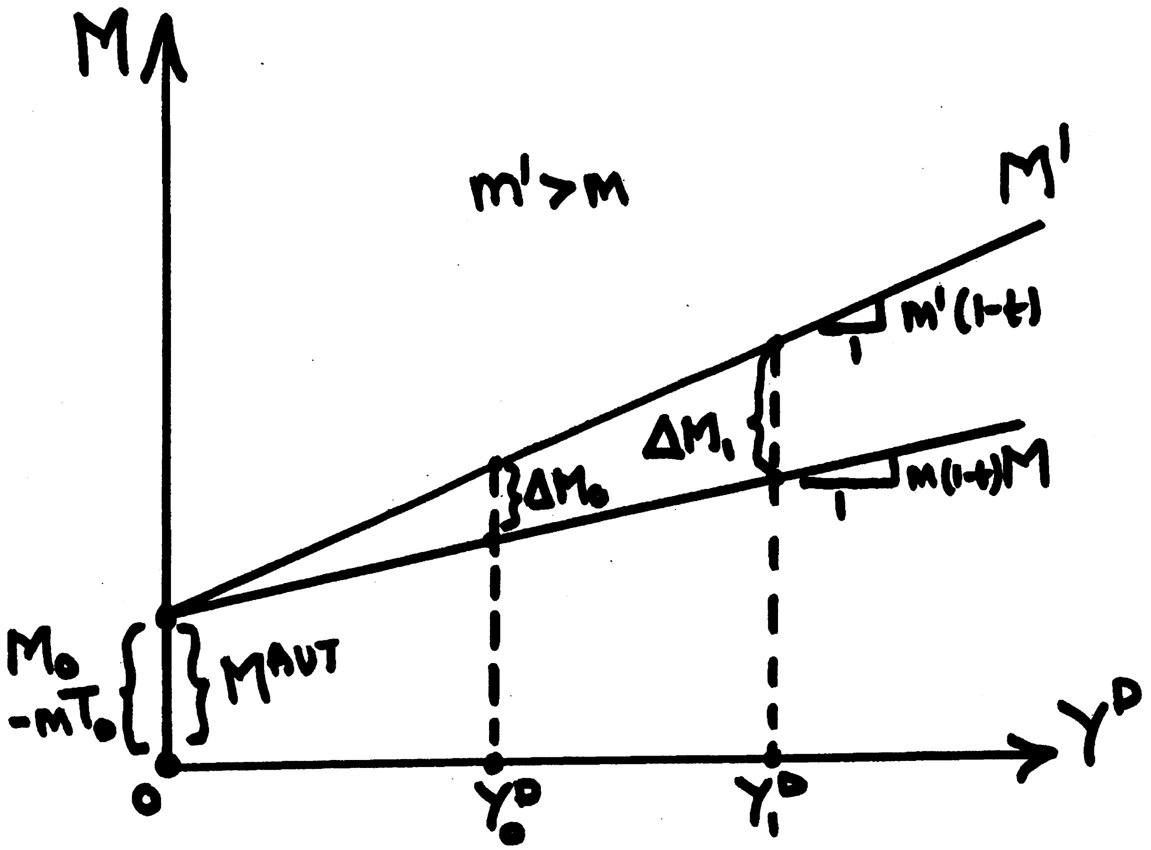
Figure 6



$\Delta M_1 > \Delta M_0$ for a given $\Delta m > 0$ $(m' > m)$ because $Y_1^D > Y_0^D$ and so the drain into the import leakage is larger the greater the level of disposable income.