MODEL 2

A CLOSED ECONOMY with a MARGINAL TAX RATE

1. Model 1 is the simplest macroeconomic model that we can construct that will allow us to ask policy questions. An economic theorist would want to see if she could construct a model that is a better approximation to the real US economy. And one that would allow her to answer more realistic policy questions.

2. One way to proceed is to adjust the tax regime to take account of the fact that tax revenues are an increasing function of income. So Model 2 will add **induced taxes**, $T^{IND} = tY$ to autonomous taxes, T_0, where **t = mpt** is the **marginal propensity to tax** (the **marginal tax rate**). So we will have to adjust our consumption function because consumption depends on disposable income and disposable income depends on taxes.

3. The new **tax function** has the algebraic form:

$$T = T(Y) = T_0 + tY \qquad T_0 < 0 \qquad 0 < t < 1.$$

The linear tax function is plotted in Figure 1. T_0 is assumed to be negative because we are assuming that T is net of transfer payments. If Y drops to a sufficiently low level then the government will pay out more in transfers to households (such as unemployment benefits and food stamps) than it will collect from them in terms of tax revenues. $\mathbf{T_0}$ is the **vertical intercept** of the tax function (VITF) and **t** -- the marginal tax rate/marginal propensity to tax -- is the **slope**

coefficient of the tax function (SCTF). We assume that t lies between zero and one. Actually this tax function is not completely unrealistic and we might choose a "ball park" value for t of .25 so that, on average, 25^c of any additional dollar received by households goes to the government. Again we have a useful *stylized fact* that the mpt is less than the short-run mpc, i.e. $0 < t < c < 1$.

4. **Disposable income** in Model 2 takes the form:

$$Y^D = Y - T(Y)$$

$$= Y - (T_0 + tY)$$

$$= Y - T_0 - tY$$

$$= - T_0 + (1-t)Y$$

so that disposable income (Y^D) is a linear function of income (Y) (see Figure 2). In this formulation (1-t)Y is the amount of any additional dollar that the households have available to purchase goods and services, C, i.e. $\Delta Y^D/\Delta Y = 1\text{-}t$.

5. Model 2's Consumption Function takes the form:

$$C = C^*(Y) = C(Y^D)$$

$$= C_0 + c\,Y^D$$

$$= C_0 + c\,(Y - T)$$
$$= C_0 + cY - cT$$

$$= C_0 + cY - c\,(T_0 + tY)$$

$$= C_0 + cY - cT_0 - ctY$$

$$= C_0 - cT_0 + cY - ctY$$

$$C = C_0 - cT_0 + c(1- t)Y \qquad C_0 - cT_0 > 0$$

$$0 < t < c <1 \Leftrightarrow 0 < c(1- t) < 1$$

$$= C^{AUT} + C^{IND}$$

where $C^{AUT} = C_0 - cT_0$ is the **vertical intercept** of the CF and $C^{IND} = c(1- t)Y$ is its **slope coefficient**. We *assume* that the vertical intercept of the CF is positive, and can *deduce* that its slope coefficient is positive but less than one given our *assumption* that c and t are both numbers between zero and one.

[$1-t > 0$ if $t < 1$; $c(1-t) > 0$ since $c > 0$ and $1-t > 0$; $1-t < 1$ if $t > 0$; and $c(1-t) < 1$ since $1-t < 1$ and $c < 1$]

The CF is plotted in Figure 3. We will refer to c(1-t) as the effective mpc or the *mpc out of income*. We will refer to c as the *mpc out of disposable income*. That is $c(1-t) = \Delta C/\Delta Y$ and $c = \Delta C/\Delta Y^D$.

6. With a new CF we also have a new AD function.

$$\begin{aligned} AD = f(Y) &= C(Y) + I(Y) + G(Y) \\ &= C_0 - cT_0 + c(1-t)Y + I_0 + G_0 \\ &= C_0 + I_0 + G_0 - cT_0 + c(1-t)Y \end{aligned}$$

$$AD = A_0 + G_0 - cT_0 + c(1-t)Y \qquad \begin{aligned} A_0 + G_0 - cT_0 &> 0 \\ 1 > c(1-t)Y &> 0 \end{aligned}$$

where $A_0 + G_0 - cT_0$ is the vertical intercept of Model 2's AD function (VIAD) and where c(1-t)Y is the slope coefficient of the AD function (SCAD). The AD function is plotted in Figure 4 where we see that the new AD function, AD^t, is **flatter** than the AD function of Model 1. This is because tax revenues now increase as income increases -- $\Delta AD^t/\Delta Y = c(1-t) < c = \Delta AD/\Delta Y$. Referring to the circular flow diagram in Figure 5 we see that taxes now reduce disposable income which in turn damps down induced AD which is simply induced consumption. There are now two leakages from the circular flow, one into induced taxes, T^{IND}, and one into saving, S. Therefore changes in autonomous expenditure lead to smaller multiplier effects -- smaller changes in Y^e.

7. The rest of Model 2 is essentially the same as Model 1 and so I leave its full development to a series of exercises (see part 2 of the Model 2 section of the Assignment 8 Key). Part (a) of the Key is the full algebraic version of Model 2. Figure 2 of the Assignment Key shows the model in diagrammatic form. And section (c) on page 4 provides the formula for the equilibrium level of income for Model 2. Section (d) on page 4 of the Key shows the multipliers for Model 2. These multipliers are derived from the basic QCS equation for this model:

$$\Delta Y^e = \frac{1}{1-c(1-t)} (\Delta A_0 + \Delta G_0 - c\Delta T_0)$$

or

$$\Delta Y^e = \frac{VIAD - VIAS}{SCAS - SCAD}$$

since VIAD = $\Delta A_0 + \Delta G_0 - c\Delta T_0$, VIAS = 0, SCAS = 1, and SCAD = c(1-t).

In order to distinguish Model 2's multipliers from those of Model 1 we attach a t superscript to them. We write k_t rather than k_t^t because the superscript seems redundant in this case -- if we are calculating the impact on Y^e of a change in the marginal tax rate, t, then we must obviously be dealing with Model 2. As is pointed out -- somewhat inelegantly -- the k_t multiplier is a pain in the nether regions to calculate using algebra although we will see later that it is easy to derive using calculus. The calculation of each of the multipliers requires us to set one or more of the autonomous/ exogenous variables equal to zero. And, of course, we are always assuming that c and t remain constant, **except when we calculate k_t.** In the case of k_t we are talking about **pivoting** the AD curve **downwards** because the higher marginal tax rate leaves households less to spend each time income increases and so Y^e and the multipliers are all reduced relative to Model 1.

8. Note that in Model 2 $|k_T^t|$ is **not** equal to $k_G - 1$ (p.5 (f)) and the k_{BB} is **not** *equal to one* although it is positive (p.5 (g)) and so balanced budget policies are still not income neutral (i.e. they alter Y^e).

9. Finally, note that the introduction of a marginal tax rate of 0.25 has reduced the size of all the multiplier effects by a large amount. For example, $k_G = 5$ if c = 4/5 but $k_G^t = 2.5$ with c =4/5 and t = ¼ (p.5 (f)).

Figure 1

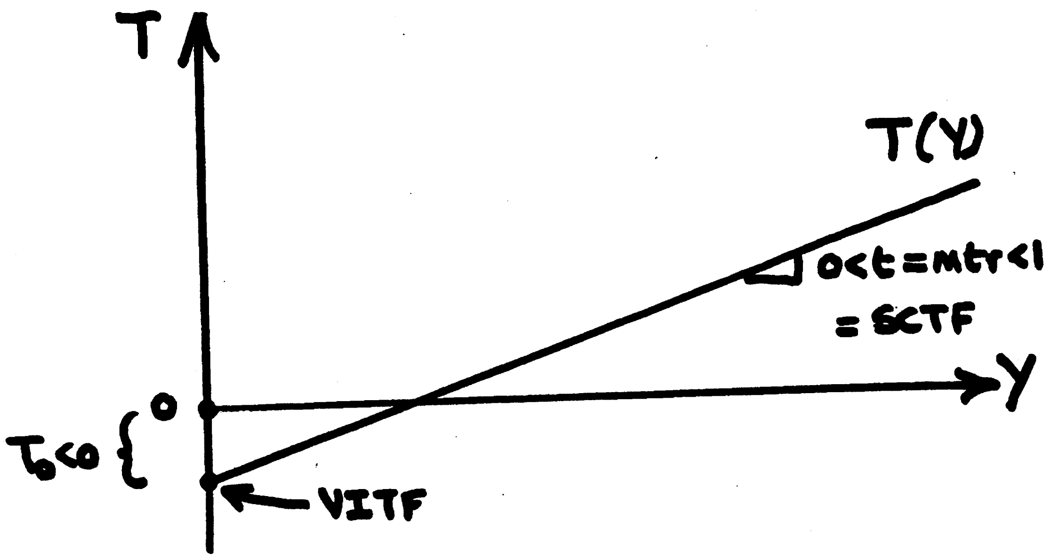

Figure 2

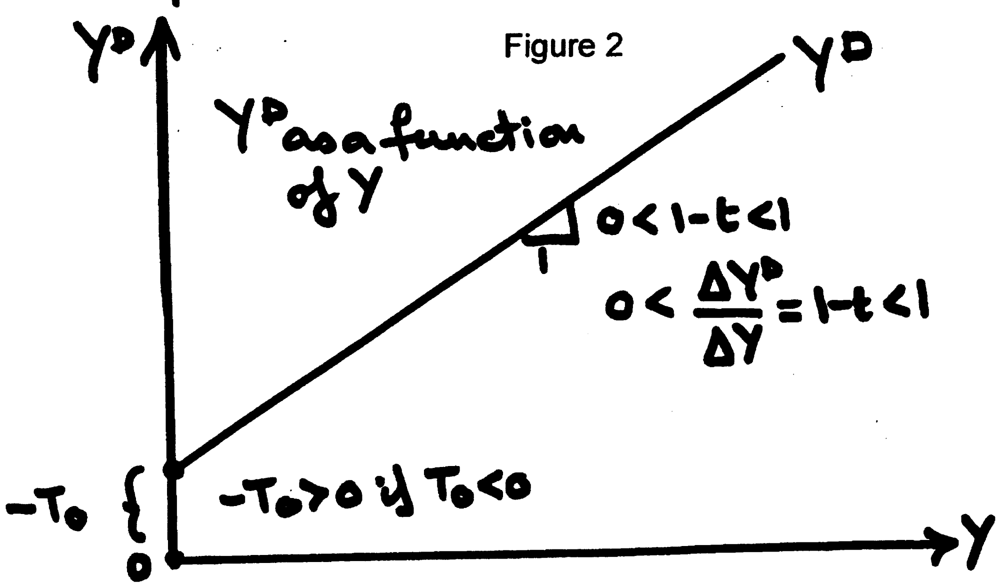

Figure 3

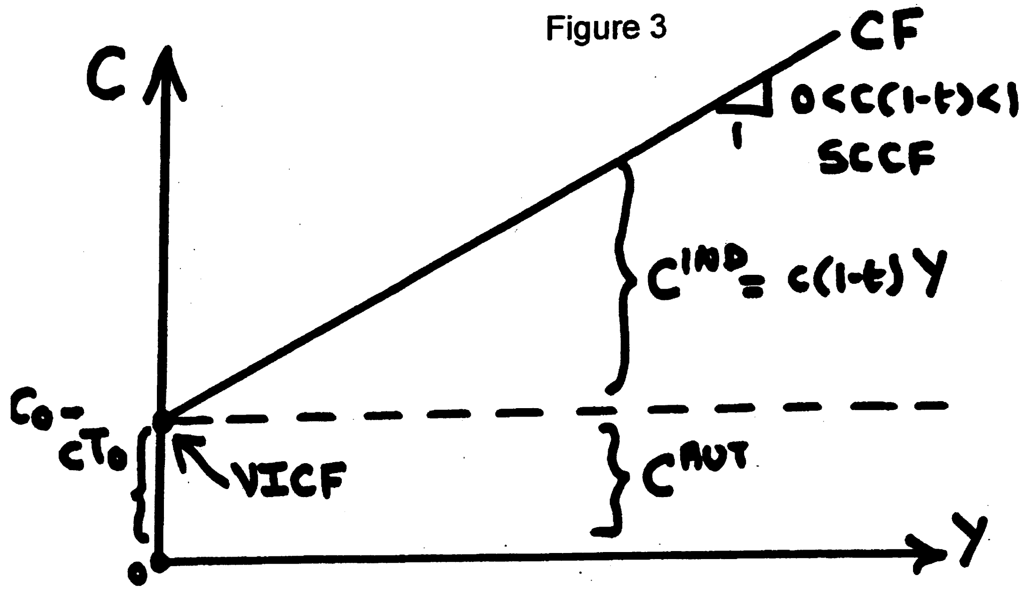

Figure 4

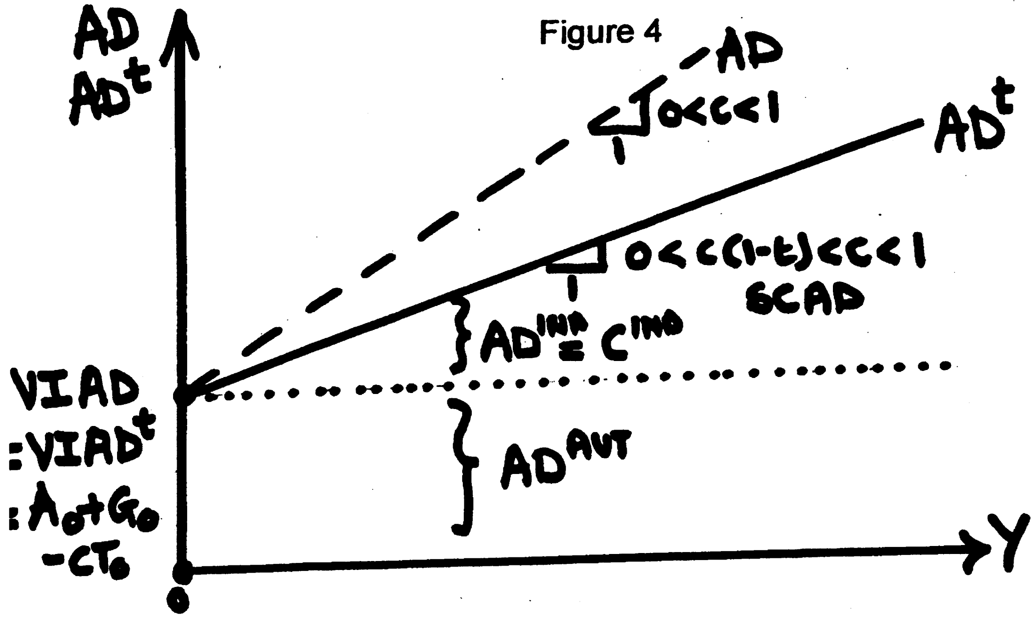

Figure 5