

MODEL I

CLOSED ECONOMY, ALL TAXES AUTONOMOUS

1. In this model there are three sets of economic agents: households, firms, and the government. How the model works is most easily understood with the aid of a “circular flow of income” diagram (see Figure 1).

Households own all of the factors of production, which they hire to the firms in exchange for income, Y . The firms use the inputs that they hire to produce output, AS . The profit maximizing firms' output level is always equal to the current level of AD . AD depends upon aggregate income, Y , and exogenous factors that determine the level of autonomous expenditure by the three sectors.

2. Households spend their income on consumer goods (C), where C is a linear function (called the *Consumption Function* (CF)) of Y . However, household expenditures are really dependent on disposable income, Y^D (see Figure 2).

Consumption consists of two components. The first component is an exogenous component, $C_0 > 0$. This exogenous consumption may be thought of as that level of consumption that households would undertake even if their incomes were zero (remember that this is a short-run model). The second component, called **induced consumption**, C^{IND} , is equal to the **marginal propensity to consume** ($mpc = c$) times disposable income ($Y^D = Y - T$). Because tax revenues (T) are exogenous/autonomous, i.e. $T = T(Y) = T_0$ (see Figure 3) the consumption function takes the algebraic form:

$$\begin{aligned}
C &= C(Y^D) = C_0 + cY^D \quad C_0 > 0, 0 < c < 1 \\
&= C_0 + c(Y - T_0) \\
&= C_0 - cT_0 + cY
\end{aligned}$$

where $C_0 - cT_0$ is autonomous consumption, C^{AUT} , the vertical intercept of the CF, and c is the slope coefficient of the CF (see Figure 4). The $-cT_0$ term arises because taxes reduce autonomous consumption but only by the mpc times T_0 since some of the tax is paid from income that would otherwise have been saved. An **increase/decrease** in C_0 will shift the CF **upwards/downwards**. An **increase/decrease** in T_0 will shift the CF **downwards/upwards** by an amount depending on the mpc.

By assumption saving is a residual -- the amount of income not consumed, i.e. $S = Y - C$. Therefore $S = Y - C_0 - cY^D = Y - C_0 - c(Y - T_0) = - (C_0 - cT_0) + (1-c) Y$ where $1-c = \mathbf{1 - mpc = mps}$ (the marginal propensity to save). Saving is a leakage from the circular flow of income in Figure 1.

3. **Induced** consumption, C^{IND} , flows from households to firms as a flow of expenditure that is part of Aggregate Demand, AD. Autonomous consumption, $C_0 - cT_0$, is also part of AD but since it is exogenous we show it in Figure 1 as flowing into the firm sector independently of income.
4. Firms' capital expenditures are described by an investment function that makes investment, I , a constant function of income (called the *investment function* (IF)). That is I is completely autonomous/exogenous and so $I = I_0$. We write the investment function as:

$$I = I(Y) = I_0 \quad I_0 > 0$$

(see Figure 5). Because I is a purely autonomous/exogenous component of AD we show it entering the model from outside the circular flow of income in Figure 1, where I_0 is added to C_0 to form **autonomous private expenditures**, A_0 . (Remember that this is a one good model in which there is no distinction between consumption and capital goods.)

5. Government expenditure on goods and services, G , is also strictly autonomous and so we write:

$$G = G(Y) = G_0 \quad G_0 > 0.$$

Remember that G does not include transfer payments such as unemployment insurance benefits (UIB).

6. Therefore AD consists of two components: autonomous/exogenous AD_0 , the sum of the three types of autonomous expenditures ($C_0 - cT_0 + I_0 + G_0$), and induced AD /expenditure which is just induced consumption, C^{IND} . We can therefore write AD as a linear function of income, the *aggregate demand function*, taking the form:

$$AD = f(Y) = C(Y) + I(Y) + G(Y)$$

$$= C_0 + cY^D + I_0 + G_0 \quad 0 < c < 1$$

$$= C_0 - cT_0 + I_0 + cY + G_0$$

$$= C_0 + I_0 + G_0 - cT_0 + cY$$

$$AD = A_0 + G_0 - cT_0 + cY \quad A_0 = C_0 + I_0 > 0 \quad A_0 + G_0 - cT_0 > 0$$

where $A_0 + G_0 - cT_0$ is the **vertical intercept** of the AD curve (VIAD) and c , the mpc, is the **slope coefficient** of the AD curve (SCAD) (see Figure 7), and where we have incorporated government transfer payments into tax revenues.

7. The macro models that we will discuss -- also referred to as **Income-Expenditure models** and **Keynesian-Cross models** -- are essentially demand driven. The supply side of the model is not developed at all. Firms are assumed to passively change their output in response to changes in AD since, *by assumption*, they cannot change prices, or order books, and do not hold inventories. Therefore the AS *function* is an **identity function** (with domain equal to the set of non-negative real numbers, R^0), with a **zero vertical intercept** (VIAS), and a **slope coefficient** (SCAS) equal to *one*. If we plot output against income then we get a forty-five degree line (i.e. a line with a slope equal to one passing through the origin) because GNP_{MP} and GNP_{FC} are just two different measures of the same magnitude. $AS = GNP_{MP} = Py$ (where we can set $P = 1$) and $GNI_{FC} = Y$ are always exactly equal to one another. So we write the AS function as:

$$AS = g(Y) = Y.$$

This is plotted in Figure 8. The vertical intercept of the graph of the AS function (VIAS) is the origin ($AS(0) = 0$) and the slope coefficient of the graph is equal to 1 ($SCAS = 1$) since changes in income and output are always equal in this and subsequent models.

8. Figure 9 puts the AS and AD graphs together on the same diagram and labels their intercepts and slopes.

9. The **equilibrium condition** for our first macro model (and all subsequent versions) requires that $AD = AS = Y^e$. Referring to Figure 10 we observe that at Y_1 , there is aggregate excess supply (EAS), i.e. $AS_1 > AD_1$. Of course, we must remember that *in our QCS world we cannot say anything about what happens in disequilibrium*, but that will not stop an economist telling a story that provides an economic rationale for how her model works.

The economist's story says that at Y_1 firms are producing more than the economy can absorb, and are paying out more in income than they receive back in the form of expenditures on the goods they propose to produce (the level of output that will maximize their profits if sold). Our firms cannot adjust prices to restore equilibrium because prices are fixed at $P=1$, and they are not allowed to adjust their order books and they do not hold inventories and so they cannot add their excess output to inventory. This allows the firms only one way to adjust: they have to cut back production -- to the level of aggregate demand AD_1 .

But, if firms produce a smaller output then they need to hire fewer inputs and so they pay out less income and so AS and Y contract one for one with the fall in AD . But if Y falls then AD will also fall (because C^{IND} will fall and so C falls and AD falls), therefore the excess aggregate supply, $EAS = AS - AD$, will continue to exist (although it will become smaller). So long as there is EAS , AS and Y will continue to fall until equilibrium is achieved at Y^e . Y_1 has no special significance, except that it is to the right of Y^e , and so Y will fall whenever Y lies above Y^e .

Alternatively we can start at Y_0 in Figure 10 where we see that $AD_0 > AS_0 = Y_0$ and so aggregate demand is greater

than the output of firms, and firms are paying out less in factor incomes than they are receiving in revenues from sales. Profit maximizing firms would want to expand output in this situation, given the restrictions that we have placed on their operations. And therefore AS will increase until it is equal to the level of AD but that means that the firms must hire more inputs and pay out more factor incomes. Therefore, the increase in AS is accompanied by an increase in Y and hence AD increases yet further (because induced consumption will increase) and so the economy is still not in equilibrium.

This expansion process will continue so long as there is excess aggregate demand, $EAD = AD - AS$. Again there is nothing special about Y_0 except that it lies to the left of Y^e , and so no income level below Y^e can be an equilibrium income level for the economy.

This means that Y^e is the only sustainable income level, where $AD = AS$. At Y^e planned output of the profit maximizing firms is equal to the actual output that they can sell, and the incomes firms pay out to factors of production are equal to the revenues that they obtain from selling their output to households, other firms, and the government sector. For the economy to be in equilibrium there must be no unsatisfied transactors, so that every economic unit that wishes to purchase goods can do so, and every firm that wishes to sell goods can find some economic unit to buy them. At Y^e we must have $AD = AS$ so that there is neither EAD nor EAS and no sector of the economy has any incentive to change its current activity level.

10. We can now write down the **formal algebraic version** of our first income determination model. We have:

$$f, g: \mathbb{R}^0 \rightarrow \mathbb{R}^0$$

$$AD = f(Y) = A_0 + G_0 - cT_0 + cY$$

$$A_0 + G_0 - cT_0 > 0 \quad 0 < c < 1$$

$$AS = g(Y) = Y$$

$$AS = AD = Y^e$$

where $c \in \mathbb{R}$. Carefully compare these equations with the equations that appear on page 91 of the Manual. Although from an *economic* point of view the two models are completely different, from a *mathematical* point of view they are just special cases of the system developed in MR 4. We can exploit this common mathematical structure to aid us in our understanding of how the models work.

11. We can **solve for the equilibrium level of income** by using the equilibrium condition. That is:

$$AS = AD$$

$$Y^e = A_0 + G_0 - cT_0 + cY^e$$

$$Y^e - cY^e = A_0 + G_0 - cT_0$$

$$Y^e (1 - c) = A_0 + G_0 - cT_0 \quad A_0 + G_0 - cT_0 > 0 \quad 0 < c < 1$$

$$Y^e = \frac{1}{1-c}(A_0 + G_0 - cT_0) > 0.$$

Alternatively we can use the general form for the solution of the x variable (the variable appearing on the horizontal axis

in the graph) for a set of two simultaneous linear equations, which is:

$$Y^e = \frac{VIAD - VIAS}{SCAS - SCAD}$$

$$= \frac{(A_0 + G_0 - cT_0) - 0}{1 - c} > 0.$$

Note that Y^e must be positive since it is the ratio of two positive numbers. However, we know that Y^e will be a relatively large positive number in reality because autonomous expenditures, $A_0 + G_0 - cT_0$, are measured in trillions of dollars and, $1/(1-c)$, the **autonomous expenditure multiplier**, is greater than one.

12. Note that because we possess certain *stylized facts* about the US economy -- the economy that we are interested in modeling -- we can get more specific information about the equilibrium income level and QCS properties of our macro model, than was the case with the very general supply and demand analysis that we undertook earlier. These stylized facts are that the AS curve has a *zero* vertical intercept and *unit* slope, and that the mpc (slope coefficient of the AD function) lies between *zero* and *one*.
13. There are only two **QCS results** that concern us. These are the effects on Y^e of changes in the two fiscal policy variables, G_0 and T_0 . Since these variables are both components of the VIAD, changes in these variables correspond to **shifts** in the AD curve. An *increase* in G_0 shifts the AD curve *upwards* (and parallel to the original AD curve because $c = mpc$, the SCAD, is held constant) and causes a *movement along* the AS curve until equilibrium is restored at Y^e_1 (see Figure 11).

The shift of the AD curve causes EAD (EAD_0) at Y^e_0 and so firms respond by hiring extra inputs (and increasing factor payments) so that they may increase AS to meet the increase in AD. But the increase in Y leads to a further increase in AD, and AS and Y continue to increase until equilibrium is achieved at Y^e_1 where $AD_1 = AS_1$. Of course, in **QCS terms** this change from Y^e_0 to Y^e_1 is *timeless* -- instantaneous if you like -- since our model is *always* in equilibrium.

What we have described is a “**multiplier**” **process** -- an expansion/contraction of Y , which is a (positive/negative) *multiple* of the change in AD, brought about by the initial increase/decrease in autonomous expenditure.

Although when doing 207 you probably referred to “the” multiplier, there are in fact **many multipliers** -- one for each endogenous variable with respect to each of the exogenous variables and the parameters of the model. In our case we are only interested in one endogenous variable, Y^e , and we will ignore the multipliers associated with the endogenous variables, C and T . (We also ignore the AS and AD multipliers since they will always be identical to the Y^e multipliers.) Further we will ignore changes in C_0 and I_0 and c , none of which are under the policy makers’ control. Therefore we will investigate only *three* multipliers -- the autonomous government expenditure multiplier, k_G , the autonomous tax multiplier, k_T , and the balanced budget multiplier, k_{BB} .

14. In order to study the QCS properties of our model we must start from its **equilibrium equation**, i.e.

$$Y^e = 1/(1-c) [A_0 + G_0 - cT_0]$$

If we change G_0 or T_0 or both then there will be a change in Y^e , i.e.

$$\Delta Y^e = 1/(1-c) [\Delta A_0 + \Delta G_0 - c\Delta T_0].$$

Now let us assume that c and ΔA_0 are held constant, i.e. $\Delta c = \Delta A_0 = 0$. (Think about the analogy with the supply and demand model and the ceteris paribus assumption and how, when we violated that assumption by changing one or more exogenous variable(s), that brought about changes in the equilibrium levels of P^e and Q^e). Then ΔY^e becomes

$$\Delta Y^e = 1/(1-c) [\Delta G_0 - c\Delta T_0].$$

This is our **basic QCS equation** that will be used over and over again in the lectures and is the key to answering the questions in Assignment 8.

15. To *calculate the government expenditure multiplier* we need to isolate the effect of the change in government expenditures from those associated with changes in autonomous taxes, T_0 . We achieve this by setting $\Delta T_0 = 0$. This reduces our basic QCS equation to

$$\Delta Y^e = 1/(1-c) [\Delta G_0] > 0.$$

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(Once again you should look for the analogy with what we did to the equations for ΔP^e and ΔQ^e when going through our QCS exercises with the supply and demand model.) This equation tells us what happens to the equilibrium level of income, Y^e , when we increase autonomous government

expenditures, G_0 -- it causes Y^e to increase. From this equation we can calculate the **government expenditure multiplier**, $\Delta Y^e / \Delta G_0 = k_G$.

The autonomous government expenditure multiplier, k_G , is equal to the change in the equilibrium level of income, Y^e , brought about by a unit (say, \$1b) change in autonomous government expenditures. This multiplier is *directly* related to the *mpc* (the slope of the AD curve, SCAD) and *inversely* related to the *mps*, where the $mpc = \Delta C / \Delta Y$ and the $mps = \Delta S / \Delta Y = 1 - c$.

It is important to be able to show that the autonomous government expenditure multiplier is larger than unity. Otherwise there would be little point in calling it a “multiplier”.

16. [The concept and terminology of the multiplier was developed in the 1920's by Richard Kahn, a Cambridge (England) economist. Kahn's idea was popularized by John Maynard Keynes (pronounced “canes”) who, like Kahn, was a Fellow of King's College, Cambridge. Keynes published “The General Theory of Employment, Interest and Money” in 1936/7. “The General Theory” is arguably the most important book on economics written in the twentieth century and the progenitor of macroeconomics. Hence the diagram in Figure 9 (which does **not** appear in “The General Theory”) is referred to as the “Keynesian” Cross.]
17. If the autonomous government expenditure multiplier is larger than unity then increasing autonomous government expenditure will cause the equilibrium level of income to increase by **more than** the change in autonomous government expenditure. The proof that $k_G > 1$ is

straightforward and is the model for most of the subsequent proofs in this section of the Manual and most of the proofs in Assignment 8.

Proof: $k_G = 1/(1-c)$.

k_G will be greater than one if its numerator is larger than its denominator. This requires us to show that $1 > 1-c$. But $c > 0$ and so $1-c$ must be less than 1. We therefore conclude that $k_G > 1$. (Notice that for $1/(1-c)$ to be a positive number $1-c > 0$ and given that $c > 0$ we are implicitly assuming that $c < 1$.)

The algebraic demonstration is shorter

$$k_G = 1/(1-c) > 1 \Rightarrow 1 > 1-c \Rightarrow 0 > -c \Rightarrow c > 0$$

(where \Rightarrow means “implies”).

So for $k_G > 1$ we require that $c > 0$ which is what we have been assuming (some part of any increase in income must be consumed so that the mpc is positive). But notice that the ratio $1/(1-c)$ is only defined if $1-c$ is not equal to zero (division by zero is an undefined mathematical operation) and we do not want k_G to be negative. So we also must assume that $c < 1$ which also agrees with what we have been doing (i.e. assuming that consumers will not increase consumption by more than the increase in their (disposable) income). So we have proved that $k_G > 1$ if, as we have assumed, $0 < c < 1$.

18. Let us assume that the mpc = 4/5 -- which is a “ball park” figure for the *short-run* mpc in the US. Then $1-c = 1 - 4/5 = 1/5$ and $1/(1-c) = 1/(1/5) = 5$ (the reciprocal of one fifth is five). Therefore if consumers, in the short-run, spend eighty

cents of every (disposable) dollar they receive then $k_G = 5$ and every extra dollar of government expenditure will (ultimately in the real world, instantaneously in our QCS world) lead to an increase in the equilibrium level of income equal to five dollars. That is $\Delta Y^e = k_G \Delta G_0 = 5 \Delta G_0$. So, for example, if the government were to increase G_0 by \$200b then Y^e would increase by \$1t, i.e. $\Delta Y^e = k_G \Delta G_0 = 5 \$200b = \$1t$. While a government multiplier of five is far too large (estimates of first and second year changes in GDP brought about by changes in government expenditure on goods and services for the US economy range between 1 and 2 with the consensus closer to 1) the basic process we have described is not too far from how we think that this type of fiscal policy operates in practice.

19. An **increase** in *autonomous taxes* will shift the AD curve **downwards** (see Figure 12), but only by *the mpc times the change in taxes* because some of the taxes will be paid for out of saving. This means **that the autonomous tax multiplier, $k_T = \Delta Y^e / \Delta T_0$, is negative.**

The *autonomous tax multiplier* is calculated using the basic QCS equation

$$\Delta Y^e = 1/(1-c) [\Delta G_0 - c\Delta T_0].$$

Where we are again assuming that A_0 and c are held constant, i.e. $\Delta A_0 = \Delta c = 0$. (Since c , the $mpc = SCAD$, is held constant we are making parallel shifts in the AD curve.) In this case we set $\Delta G_0 = 0$ and the **basic QCS equation** now becomes

$$\Delta Y^e = 1/(1-c) [-c\Delta T_0].$$

or

$$\Delta Y^e = -c/(1-c) [\Delta T_0].$$

Therefore the **autonomous tax multiplier is a negative number**, $k_T = \Delta Y^e / \Delta T_0 = -c/(1-c) < 0$ and therefore **must** be smaller than the autonomous government expenditure multiplier, since every negative real number is smaller than every positive real number (and zero, of course). But what we are really interested in is the relative “bang for the buck” that we get from each type of policy change, so we should really compare the *absolute value* of k_T with k_G . [The **absolute value of a real number** is its numerical value, ignoring its sign. That is, the absolute value of a , in symbols $|a|$ (where $a \in \mathbb{R}$) is equal to a if a is non-negative (positive or zero) and $-a$ if a is negative.] So $|k_T| = -k_T = c k_G < k_G$ since $c < 1$ (where $k_T = -c/(1-c)$). This calculation is done at the foot of Figure 12.

20. We can therefore deduce that, dollar for dollar, government expenditures have a larger expansionary effect on the economy than autonomous taxes have a contractionary effect on the economy. Indeed, if we again assume that the $mpc = 4/5$ so that $k_G = 5$, then $k_T = -ck_G = -(4/5)5 = -4$, and $|k_T| = -(-4) = 4 < 5$. And so an *increase* in $T_0 = \$200b$ will cause Y^e to *decrease* by $\$800b$, i.e. $\Delta Y^e = k_T \Delta T_0 = -4 \$200b = -\$800b$.
21. At this point an economic theorist would be interested to see if $|k_T|$ is **always** equal to $k_G - 1$ or whether this is simply the result of choosing an mpc of $4/5$. The answer is that *the result is true for this particular model*.

The **proof** is straightforward:

$$k_G - 1 = 1/(1-c) - 1 = 1/(1-c) - (1-c)/(1-c)$$

$$= (1 - 1 + c)/(1-c) = c / (1-c) = |k_T|.$$

Therefore we know that if $c = 9/10$ then $k_G = 10$, $k_T = -9$ and $|k_T| = 9 = k_G - 1$, and if $c = 1/2$ then $k_G = 2$, and $k_T = -1$ and $|k_T| = 1$. Obviously $k_t = - (k_G - 1)$ -- if this *isn't* obvious then you missed something in the above exposition.

22. What happens if we **simultaneously** change *autonomous government expenditure and autonomous taxes*? This is the **balanced budget multiplier** case. We must start from our basic QCS equation

$$\Delta Y^e = 1/(1-c) [\Delta A_0 + \Delta G_0 - c\Delta T_0].$$

Once again we set $\Delta A_0 = \Delta c = 0$ which yields

$$\Delta Y^e = 1/(1-c) [\Delta G_0 - c\Delta T_0].$$

And so the change in the equilibrium level of income, Y^e , consists of two effects: the effect of the change in autonomous government expenditures times the autonomous government expenditure multiplier, plus the effect of the change in autonomous taxes times the autonomous tax multiplier. That is:

$$\Delta Y^e = \underset{+}{1/(1-c)} [\underset{+}{\Delta G_0}] + \underset{-}{-c/(1-c)} [\underset{+}{\Delta T_0}].$$

$$\Delta Y^e = k_G \Delta G_0 + k_T \Delta T_0.$$

Obviously the magnitude of the change in Y^e depends on the relative sizes of the changes in autonomous government expenditures and in autonomous taxes. A case that is particularly interesting arises when $\Delta G_0 = \Delta T_0$ -- the so called “balanced budget” fiscal policy. In this case the government pays for every dollar of expenditures by raising a dollar of autonomous taxes. Because the government budget (GB) is simply the difference between its revenues (GR) and its expenditures (GE), this means that this fiscal change will not affect the size of the national debt, i.e. $GB = GR - GE = T_0 - G_0$ and $\Delta GB = \Delta G_0 - \Delta T_0 = 0$ if $\Delta G_0 = \Delta T_0$.

23. Does this policy affect the equilibrium level of income? You might argue that since every dollar of government expenditure is matched by raising a dollar of taxes Y^e will not change. In other words: what the government is adding to the circular flow of income in terms of additional expenditures it is taking away again in additional taxes. However, a little thought will convince you that this type of fiscal policy is **not** income neutral, i.e. this type of policy will cause Y^e to change. This is not something that many “pols” seem to be aware of! In particular, if the economy is already at full employment then balanced budget fiscal changes will either cause *inflation* (if the $\Delta G_0 = \Delta T_0 > 0$) and *recession* (if the $\Delta G_0 = \Delta T_0 < 0$).
24. We will now **prove** that the balanced budget multiplier, $k_{BB} = \Delta Y^e / \Delta G_0 \mid (\Delta G_0 = \Delta T_0) = 1$. [The original proof was published by the great Norwegian Nobel economist, Trgve Haavelmo, in *Econometrica* in 1941.]

$$\Delta Y^e = \underset{+}{1/(1-c)} [\underset{+}{\Delta G_0}] + \underset{-}{-c/(1-c)} [\underset{+}{\Delta T_0}].$$

$$\Delta Y^e = k_G \Delta G_0 + k_T \Delta T_0$$

but $\Delta G_0 = \Delta T_0$ which means that we can replace ΔT_0 by ΔG_0 and vice versa -- that is what the equality sign means! So we can write the last equation as

$$\Delta Y^e = k_G \Delta G_0 + k_T \Delta G_0$$

or

$$\Delta Y^e = 1/(1-c) [\Delta G_0] + -c/(1-c) [\Delta G_0]$$

which means that we can factor out the common ΔG_0 to obtain

$$\begin{aligned} \Delta Y^e &= \{1/(1-c) + -c/(1-c)\} [\Delta G_0] \\ &= \{(1-c)/(1-c)\} [\Delta G_0] \\ &= \Delta G_0 (= \Delta T_0). \end{aligned}$$

Hence the balanced budget multiplier in this model is equal to one:

$$k_{BB} = \Delta Y^e / \Delta G_0 \mid (\Delta G_0 = \Delta T_0) = 1.$$

A more direct proof is to see that $k_{BB} = k_G + k_T$ (where $\Delta G_0 = \Delta T_0$) so that $k_{BB} = 1/(1-c) + -c/(1-c) = (1-c)/(1-c) = 1$.

[Can you provide an economic explanation of **why** the k_{BB} is exactly **one** in this model?]

25. A common misconception about fiscal policy that students seem to acquire from their 207 courses and textbooks is that when the economy is in a *deflationary gap* situation ($AS(Y^F) > AD(Y^F) \Leftrightarrow Y^e < Y^F$) then the government *must* increase government expenditure and/or decrease taxes. Similarly when the economy is suffering from an *inflationary gap* ($AD(Y^F) > AS(Y^F) \Leftrightarrow Y^e > Y^F$) then the government *must* decrease government expenditure and/or raise taxes.

But this is not what our model tells us. Since $\Delta Y^e = k_G \Delta G_0 + k_T \Delta T_0$ **any** combination of changes in ΔG_0 and ΔT_0 that causes Y^e to increase ($\Delta Y^e > 0$) will move the economy towards full employment. We just have to adjust the magnitudes of the changes in government expenditure and autonomous taxes to bring about the desired change in Y^e . For example, say there is a deflationary gap and that $Y^F - Y^e = \$1.1t$, and assume that our $mpc = 4/5$ so that $k_G = 5$ and $k_T = -4$. Then, say President Shrub decides to *decrease* government expenditure by \$100b because he wishes to reduce the size of the government sector and get rid of “bureaucratic fat”. The $\Delta G_0 = \$100b$ moves the economy **away** from full employment and the equilibrium level of income would *fall* by \$500b making the income gap increase to \$1.6t, **if** that was the only action taken by the administration. But, President Shrub is also keen on a tax cut and so he proposes that T_0 be cut by \$400b ($\Delta T_0 = -\$400b$). Given that the tax multiplier in our case is -4 , the tax cut will produce a fiscal stimulus equal to $\$1.6t = (-4) \cdot (-\$400b) = \$1.6t$, just sufficient to remove the income gap. So President Shrub can have his cake and eat it! This is because we have *two* fiscal policy instruments ($\Delta G_0, \Delta T_0$) and only *one* fiscal target, Y^e . So in our model we have one policy degree of freedom to play with and we could take on

another target if we wished, e.g. reducing the government deficit. In a situation in which we have one target and two instruments to achieve that target there are an *infinite* number of policy choices available to us.

26. You should be able to show that a Democratic administration could propose to increase government expenditure by \$400b. This would cause income to overshoot by \$400b (i.e. $5(\$400b) = \$2t$ which is greater than \$1.6t by \$400b). But so long as they were willing to increase taxes (say on the top 5% of household incomes) by \$100b, the **net** effect of their policies would be to close the deflationary gap. And you should be able to show that the government has similar flexibility to deal with an inflationary gap.
27. Those of you who have stayed awake this far -- and I nodded off myself several times while writing this -- will have noticed that we actually have other fiscal policies available to us. Since the balanced budget multiplier is equal to one, in this model

$$\Delta Y^e = \Delta G_0 = \Delta T_0$$

and therefore the government can remove a \$1.6t income gap by increasing government expenditure by \$1.6t so long as it raises taxes by \$1.6t too.

28. We can calculate the required change in, say, government expenditure necessary to remove an income gap of \$2t if we know the size of k_G . If $k_G = 5$ then $\Delta G_0^* = \Delta Y^* / k_G = \$2t / 5 = \$400b$, where ΔY^* is the size of the income gap, i.e. $\Delta Y^* = Y^F - Y^e$.

In general we have

$$\Delta Y^* = k_G \Delta G_0^* + k_T \Delta T_0^*$$

and we can rearrange this as

$$k_G \Delta G_0^* = \Delta Y^* - k_T \Delta T_0^*$$

or

$$\Delta G_0^* = \Delta Y^* / k_G - (k_T / k_G) \Delta T_0^*$$

which shows that there is a linear relationship between ΔG_0^* and ΔT_0^* (given k_G , k_T , and ΔY^*). This means that there are as many combinations of ΔG_0^* and ΔT_0^* that lead to ΔY^* as there are points on the straight line, i.e. an infinite number of possible policy combinations (see Figure 13).