ECONOMIC THEORY 4

SUPPLY AND DEMAND: THE ALGEBRAIC MODEL

THE BASIC ALGEBRAIC SUPPLY AND DEMAND MODEL

(1) $f,g : R^0 \rightarrow R^0$

$$Q^d = f(P) = a + bP \qquad a > 0 \qquad b < 0$$
$$Q^s = g(P) = c + dP \qquad c < 0 \qquad d > 0$$
$$Q^s = Q^d = Q^e \qquad ad - bc > 0 \Leftrightarrow -a/b > -c/d$$

where a, b, c, d $\in$R.

This is just the standard algebraic system of two linear simultaneous equations but with x and y given economic interpretations and with restrictions on the coefficients to guarantee meaningful economic solutions.

(2) We can *solve for* P^e by substituting the demand and supply equations into the equilibrium condition that yields:

$$c + dP^e = a + bP^e$$

$$dP^e - bP^e = a - c$$

$$(d-b)P^e = a - c$$

$$P^e = \frac{a-c}{d-b}.$$

$P^e > 0$ given that $a > c$ and $d > b$. (Give both a geometric and an economic interpretation of this result).

Substituting P^e into the demand equation gives

$$Q^d = Q^e = a + b\,P^e$$

$$= a + b\,\frac{(a-c)}{d-b}$$

$$= \frac{a(d-b)}{d-b} + \frac{b(a-c)}{d-b}\,.$$

Therefore $$Q^e = \frac{ad - ab + ab - bc}{d-b}$$

i.e. $$Q^e = \frac{ad - bc}{d-b} \Leftrightarrow \frac{(-a/b) - (-c/d)}{1/d - 1/b} \quad \text{[Multiply by (-1/bd)]}$$

where $Q^e > 0$ if $ad > bc$. (Give both a geometric and an economic interpretation of this result.) (Show that substitution into the supply equation yields the same solution value for Q^e.)

[Figure 1 goes here.]

(3) Notice that *Q* is on the **vertical** axis and *P* on the **horizontal** axis. This corresponds with the **algebra** of our model and we will adopt the rule that **whenever we are doing mathematics we will always follow the mathematicians' conventions**.

(4) An **increase** in demand is modeled as $\Delta a > 0$ with the demand curve shifting (parallel) **upwards** and to the **right**. A supply **increase** is associated with $\Delta c>0$ and an **upward** and **leftward** movement of the supply curve.

(5) Confirm that this system behaves exactly like our conventional Marshallian diagram. Draw a diagram with P on the vertical axis and Q on the horizontal axis and label. (Figure 2 goes here.)

Figure1

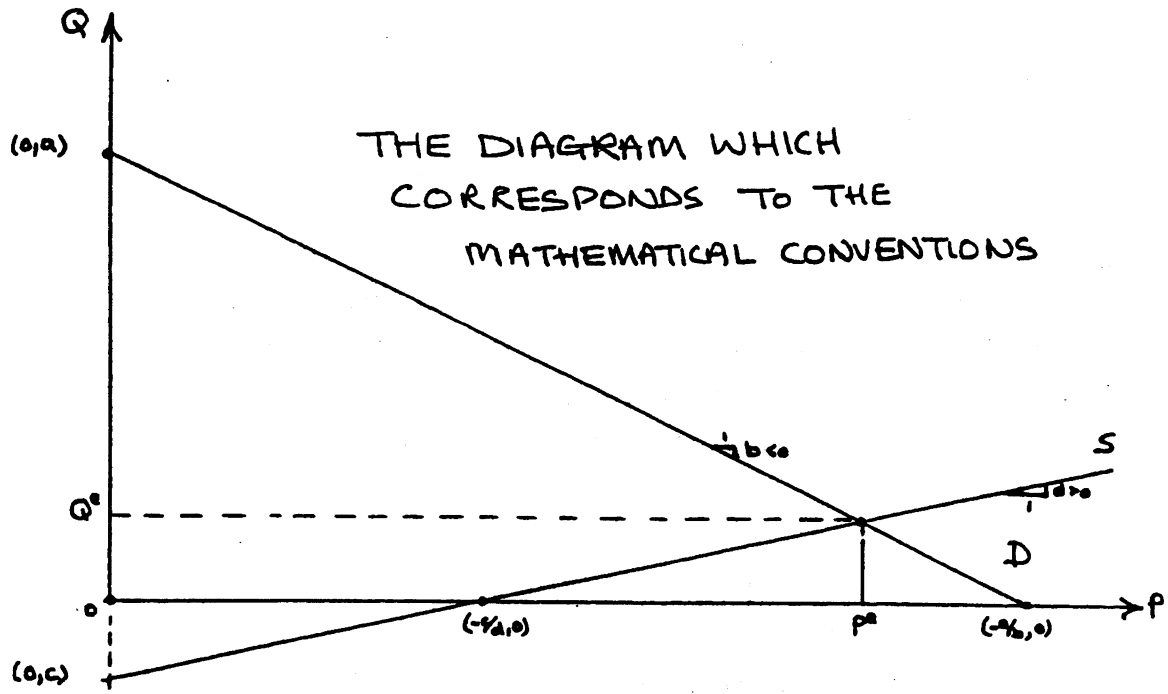
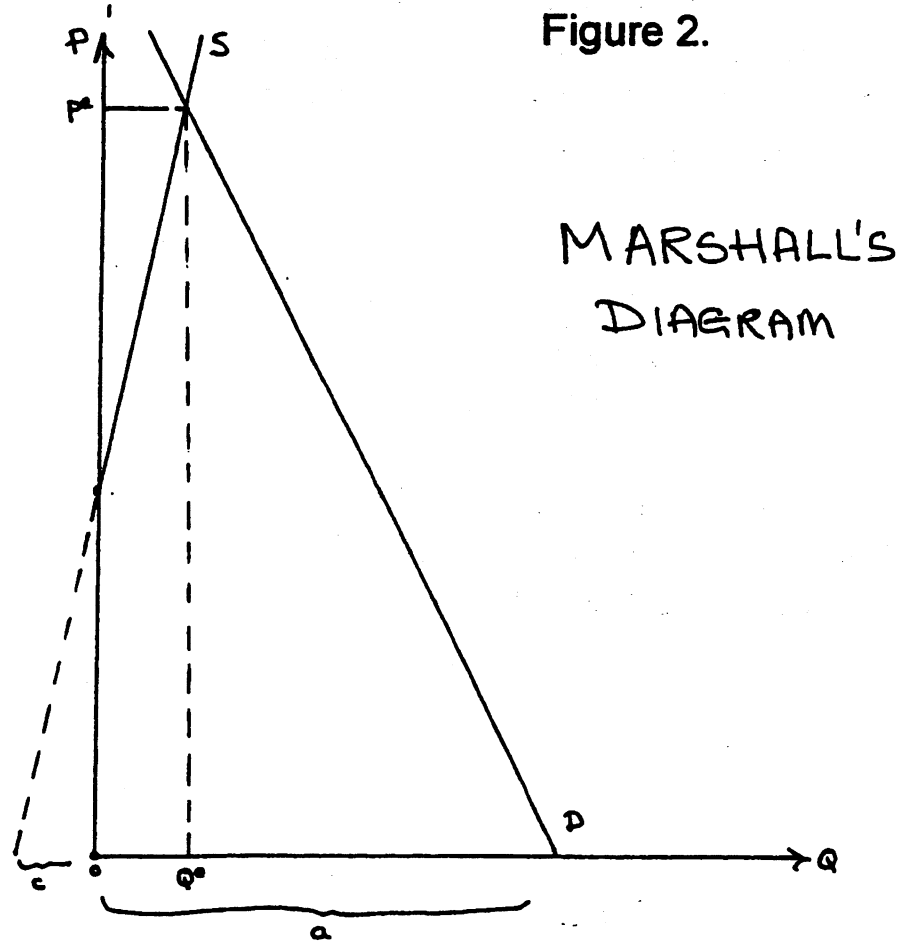


Figure 2.



ECONOMIC THEORY 5

SUPPLY AND DEMAND: QUALITATIVE COMPARATIVE STATICS

1. INTRODUCTION

Economists distinguish between **statics** and **dynamics**. In **static** analysis we assume that the system is always in *a state of equilibrium*, where the forces acting on the system are exactly balanced, for example, demand and supply are equal. In **dynamic** analysis we study the system in *motion*: either along an *equilibrium path* (a steady state or balanced growth path) or in a *disequilibrium state* where, for example, supply and demand forces are not equal and we have situations of excess demand or excess supply.

In this course, and in the majority of your economics courses, we confine ourselves to statics, or, rather, to Qualitative Comparative Statics (QCS). QCS involves **comparisons** between static **equilibria** -- we *compare* how the new *equilibrium* configuration *differs from* the *initial equilibrium*, e.g. does it have a higher price but a smaller quantity transacted relative to the initial equilibrium? In QCS we investigate the effects of **parameter shifts** on the values of the **dependent (endogenous) variables**, e.g., what happens to equilibrium price and quantity transacted if there is an increase in demand? Although the technique is extremely powerful you need to be aware of its **limitations**.

Strictly speaking when we are doing QCS all references to *disequilibrium* phenomena are *meaningless*. For example, when I say that: "because demand increased we now have a state of excess demand and therefore consumers (who are willing to pay

more for each and every quantity, i.e, their **demand prices**, P^d, have risen) will bid up the price (which will stimulate the quantity supplied while inhibiting the quantity demanded and thus reduce the excess demand) until the excess demand is eliminated," these statements are *technically meaningless* in the context of QCS, because they refer to states and processes which are not taken account in our model which assumes that the market is **always** in **equilibrium**.

Similarly we **assume** that our equilibria are **stable**, i.e., that when the system is disturbed it returns to the original equilibrium or moves to a new equilibrium in a finite time interval. A ball bearing and a round-bottomed metal cooking bowl neatly demonstrate the distinction between stable and unstable equilibria. If we place the ball bearing **inside** the bowl (when the bowl is resting on its bottom) then the ball bearing will achieve a state of rest at the bottom of the bowl and that state will be restored for (small) displacements so we say that the original equilibrium is *stable*. However, if we invert the bowl and balance the ball-bearing on its bottom (a position of rest/equilibrium) then even **very** small displacements of the ball-bearing will lead to trajectories which do not lead back to equilibrium, so we say that the original equilibrium is *unstable*. (The terminology, and indeed most of the analysis, is borrowed from classical mechanics, a branch of physics.)

We cannot say *anything* about time **paths** or **speeds** of adjustment.

Since we are doing **qualitative** comparative statics we only have qualitative information concerning parameters (i.e., we know their **signs** but not their magnitudes) and so we cannot say **by how much** the dependent variables (P^e, and Q^e) change between equilibria.

2. SUPPLY AND DEMAND – SINGLE SHIFTS

The **position** of the **demand** curve depends on: **prices** of **substitutes** and **complements**, **real income**, **tastes,** and **the number of buyers**. The **position** of the **supply** curve depends on: **input prices**, **technology**, **government variables** such as *taxes/tariffs/subsidies/regulations*, and in the case of agricultural commodities **weather** and **crop** conditions, and **the number of suppliers**.

An **INCREASE IN DEMAND** -- a **SHIFT** in the demand curve ($\Uparrow$ D) -- i.e. an increase in the quantity demanded at each and every price -- is associated with an increase in the vertical intercept parameter, **a**, i.e., $\Uparrow D \Rightarrow \Delta a > 0$. An *increase in demand* means that consumers are willing to *buy more at each and every price* and are *willing to pay more for each and every quantity*, i.e., $\Uparrow D \Leftrightarrow \Delta a > 0 \Leftrightarrow \uparrow Q^d$ and $\uparrow P^d$ ($\uparrow P^d$ stands for an increase in the demand price at each and every quantity -- where the *demand price,* P^d, is the maximum price the consumer will pay for a given quantity of the good).

[Figure 1 goes here.]

(You need to check that you know **why** an *increase* in *supply* is associated with a **leftward** shift of the supply curve in this mathematical co-ordinate system, and why $\Delta c>0$ means that supply increases.)

We have seen that $P^e = (a-c)/(d-b)>0$ and $Q^e = (ad-bc)/(d-b)>0$. If we confine ourselves to **parallel shifts** in demand and supply curves (i.e., $\Delta b = \Delta d = 0$) then the algebra of our QCS is straightforward while we lose almost no analytical insights.

EXAMPLE $\Uparrow D \Rightarrow \Delta a > 0$ where $\Delta b = \Delta c = \Delta d = 0$ (ceteris paribus - other things being equal -- in this case means that the supply curve does not shift and both curves retain their original slopes).

Now $P^e = (1/(d-b))(a-c)$ and so if we change **anything** on the RHS of the equation we must change the LHS, i.e.,

$$\Delta P^e = (1/(d-b))\ (\Delta a - \Delta c).$$

But $\Delta c = 0$, **in our case**, therefore

$$\Delta P^e = (1/(d-b))\ (\Delta a).$$

We wish to **sign** ΔP^e, i.e., we wish to answer the question: has P^e risen, fallen, or remained unchanged after the increase in demand? We know that $d > 0$, $b < 0$, (and therefore $-b > 0$) so that $d-b > 0$, and $1/(d-b) > 0$. We also know that $\Delta a > 0$. We therefore conclude that $\Delta P^e > 0$, i.e.,

$$\Delta P^e = \underset{+}{(1/(d-b))}\ \underset{+}{(\Delta a)} > 0.$$

The shift in demand at P_0 causes excess demand. The unsatisfied consumers who are willing to pay more (we know that an increase in demand means $\uparrow P^d$) bid up the price, so that the new equilibrium price is higher than the old equilibrium price. (Show that $\Delta Q^e = (d/(d-b))(\Delta a) > 0$, verbalize your result, and check that you have agreement with your diagrams).

3. SIMULTANEOUS SHIFTS IN DEMAND AND SUPPLY

The previous exercise was, basically, what you learned in 206, but using Marshall's diagram with the axes the "wrong" way around. Unfortunately, the real world seldom presents us with simple situations where one curve stands still while the other shifts. As economics majors you must be able to deal with simultaneous shifts in demand and supply. (At present our shifts can only be parallel since we need more technique to deal with pivots of the curves and we do not get much in the form of extra insights into the working of the model if we allow the curves to pivot as well as shift).

In general we have

$$\Delta P^e = \underset{+}{[1/(d-b)]}(\underset{+}{\Delta a}-\underset{-}{\Delta c}) > 0$$

and

$$\Delta Q^e = \underset{+}{[1/(d-b)]}\ (\underset{+}{d\Delta a}-\underset{-}{b\Delta c})$$

which is indeterminate.

Notice that in the equation for $\mathbf{\Delta P^e}$ the terms *multiplying* Δa (+1) and Δc (-1) are of **opposite sign**, whereas in the case of $\mathbf{\Delta Q^e}$ they are of the **same sign (**i.e. $d>0$ and $-b>0$). This means that **if** *the demand and supply curves* **shift** in the **same direction** (i.e., *sign* Δa = *sign* Δc) **then** we can predict what will happen to the **quantity transacted ($\mathbf{Q^e}$)**, but that the *sign* of ΔP^e will be **indeterminate**. Whereas if the curves **shift** in **opposite directions** (i.e., *sign* $\Delta a \neq$ *sign* Δc) we can *sign* $\mathbf{\Delta P^e}$ but ΔQ^e will be **indeterminate**. The economics of these results should be quite clear: an increase in demand **and** an increase in supply

means that **both** firms and households wish to *transact larger quantities*. However, households are willing to pay **higher** prices while firms are willing to accept **lower** prices. Since households and firms *agree* that Q should rise, $\mathbf{\Delta Q^e > 0}$, but since they *disagree* about price, $\mathbf{\Delta P^e}$ **is indeterminate**. On the other hand, when demand increases and supply decreases the transactors agree about price (they both wish it to rise) but disagree about the amount to be transacted. *(For the exams you need to be able to do all **four** cases).*

EXAMPLE: $\Uparrow D$ and $\Downarrow S$ (say, because real income is rising and our good is a normal good, and because wages have increased, raising costs) then

$$\Uparrow D \Leftrightarrow \Delta a > 0 \text{ and } \Downarrow S \Leftrightarrow \Delta c < 0.$$

Therefore, $\Delta P^e = (1/(d-b))\,(\Delta a - \Delta c) > 0$,

$$\qquad\qquad + \qquad + \;\; - \;\; -$$

$$\text{but } \Delta Q^e = \frac{1}{d-b}\,(d\Delta a - b\Delta c) \text{ which is indeterminate.}$$

$$\qquad\qquad + \;\; + + \;\; - \;\; - -$$

(Note that what happens to Q^e depends on **both** the **magnitudes of the shifts** Δa, Δc **and** the **slopes** of the demand and supply curves, b and d.) **All** new equilibria lie **within** the cross - hatched region (in Figure 2) delimited by the original demand (D) and supply (S) curves and the horizontal axis. **At** the new equilibrium P^e is higher than P_0^e ($\Delta P^e > 0$) but we cannot say anything about Q^e ($\mathbf{\Delta Q^e}$ **is indeterminate**).

[Figure 2 goes here.]

(Do *not* illustrate your results with single shifts in the demand and supply curves!)

4. **QCS ASSUMPTIONS**.

1. We start in equilibrium.

2. The demand curve is linear and continuous,

$$f: R \rightarrow R \quad Q^d = f(P) = a + bP .$$

3. The demand curve has a negative slope, $b < 0$.

4. The supply curve is linear and continuous,

$$g: R \rightarrow R \quad Q^s = g(P) = c + dP.$$

5. The supply curve has a positive slope, $d > 0$.

6. The vertical intercept of the demand curve lies above the vertical intercept of the supply curve, $a > c$.

7. The horizontal intercept of the demand curve is to the right of the horizontal intercept of the supply curve, $-a/b > -c/d$ or $ad-bc > 0$ or $ad > bc$.

8. All shifts of the demand and supply curves are parallel shifts.

9. We end in equilibrium. This assumes that the model is **stable** so that when we make a parameter shift the model moves instantaneously to the new equilibrium. It is interesting to note that to establish the stability of the model we have to move outside the bounds of static analysis, as was first shown by Professor Paul Samuelson who coined the term "the Correspondence Principle" to describe this interconnection between meaningful QCS and stable models. Samuelson's Nobel Prize was awarded specifically

for his pioneering work on the interconnections between statics and dynamics.

10. We have only *qualitative information* concerning the parameters and their shifts.

11. Only one of the demand and/or supply exogenous variables changes, i.e. we invoke the ceteris paribus assumption when doing each of our comparative static exercises.

You do **not** need to commit these assumptions to memory.

5. **PARTIAL EQUILIBRIUM.**

Finally, note that we are only doing **partial equilibrium analysis,** not general equilibrium analysis. For example, we do not take into account the effects of changes in the price and quantity variables in *this* market on related markets -- those for substitutes and complements to this good -- nor do we take account of any income effects via input markets. So if we are analyzing the market for **tea** and we assume that the price of coffee increases then we shift the demand curve for tea since it is reasonable to assume that tea and coffee are substitutes. The increase in the demand for tea will cause the price of tea to increase. But if coffee is a substitute for tea then tea must be a substitute for coffee, and therefore the increase in the price of tea will cause the demand for coffee to increase, and so on. In general equilibrium analysis we would want to model **both markets,** but in partial equilibrium analysis the price of coffee would be taken as *exogenous* and the feedback from the increase in the price of tea would be ignored. The issue of which variables are to be treated as **endogenous** – and are to be explained by our model – and which variables are to be taken as **exogenously** given is one of

the most important in the process of modeling an economic problem.

In the last half-century economists have increasingly concentrated on general equilibrium analysis, obtaining both many new theoretical insights into the workings of the economy and new ways of handling policy issues. However, to be able to follow, or do, this type of analysis we would need much more math than is appropriate for this course. (Assignment 8 gives you a taste for the simplest type of model we might use in studying an economic system with more than one sector.)

Figure 1

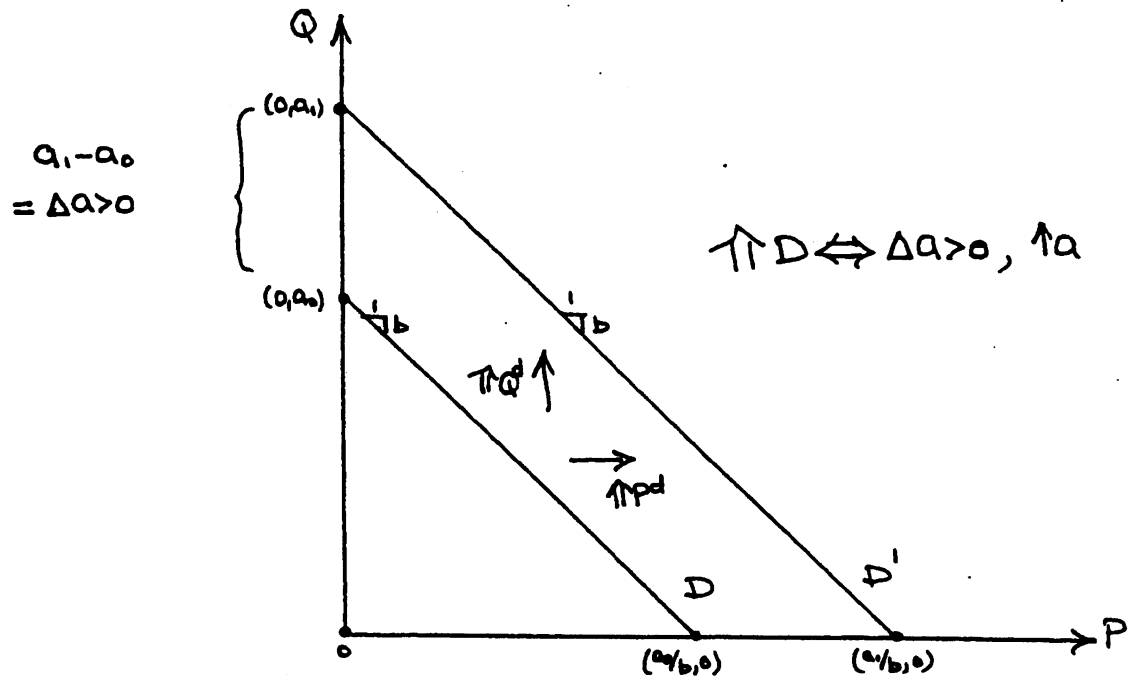
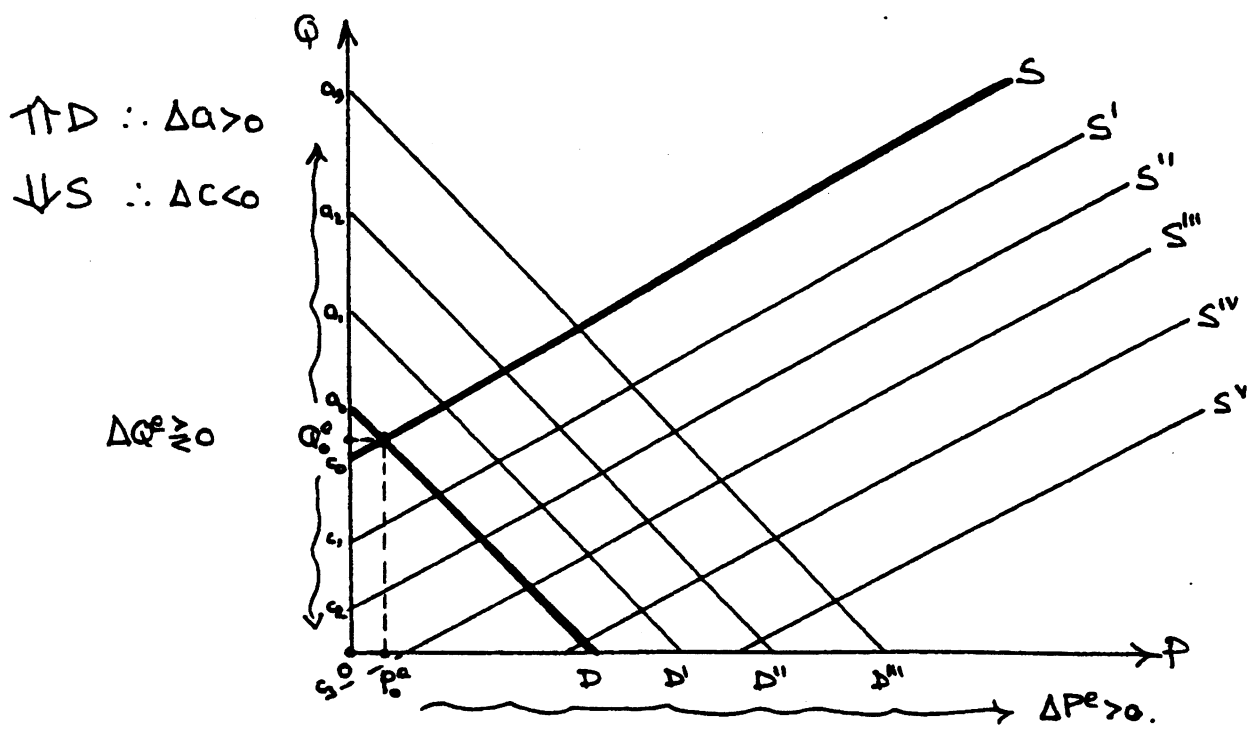


Figure 2



ECONOMIC THEORY 6

SUPPLY AND DEMAND: THE EXOGENOUS VARIABLES

1. We have been writing our supply and demand models with only two endogenous variables, price and quantity transacted. What has happened to the exogenous variables? The answer is that we have collapsed them into the two vertical intercept terms, a and c.

2. $Q^d = f(P) = a + bP$ is really a shorthand version of the true demand function, $Q^d = f(P) = a_sP_s + a_cP_c + a_lI + a_tT + a_{nb}NB + bP$, and $Q^s = g(P,\ldots) = c + dP$ is really a shorthand version of the true supply function, $Q^s = g(P,\ldots) = c_iP_i + c_{tch}TCH + c_{gov}GOV + c_wW + c_{ns}NS + dP$. Notice that all of the *a* and *c* parameters are constants, as are the exogenous variables, and so their products are also constants, and so it is perfectly permissible to replace them by **a** and **c** respectively, i.e. **a** is simply the sum of the first five terms in the true demand function -- $\mathbf{a} = a_sP_s + a_cP_c + a_lI + a_tT + a_{nb}NB$, and **c** is similarly the sum of its individual terms --- $\mathbf{c} = c_iP_i + c_{tch}TCH + c_{gov}GOV + c_w W + c_{ns}NS$.

3. We can use this knowledge to shift the demand and supply curves whenever an exogenous variable changes using the facts that $\Delta a/\Delta a_s > 0$, $\Delta a/\Delta a_c < 0$, $\Delta a/\Delta a_l > 0$ for normal goods and $\Delta a/\Delta a_l < 0$ for inferior goods, $\Delta a/\Delta a_t > 0$, $\Delta a/\Delta a_{nb} > 0$, where $\Delta a/\Delta a_s = \Delta Q^d/\Delta P_s$ etc., and that $\Delta c/\Delta c_i < 0$, $\Delta c/\Delta c_{tch} > 0$, $\Delta c/\Delta c_w > 0$, $\Delta c/\Delta c_{nb} > 0$, and that $\Delta c/\Delta c_{gov}$ takes different signs depending on the particular government variable concerned -- taxes (-), subsidies (+), and regulations (-) -- where $\Delta c/\Delta c_i = \Delta Q^s/\Delta P_i$ etc.

ECONOMIC THEORY 7

SOME ASSUMPTIONS UNDERLYING THE INCOME DETERMINATION MODEL

1. The economy produces a single good that can be used both for consumption and for capital accumulation. We refer to such economies as Ricardian "corn" (wheat) economies or as "schmoo" economies. This assumption means that we do not have to worry about relative prices or separate production sectors for consumption and capital goods. The good is infinitely malleable and can be used either for consumption (wheat into flour into bread) or for investment (seed germinates and produces a new crop of wheat next year).

2. The aggregate price level, P, is held constant and set equal to one, i.e. P=1. So in these models there is no distinction between real and nominal magnitudes and firms will **not** raise prices if there is an increase in demand for their output. Alternatively we could think of our economy as producing many goods and services whose relative prices are **fixed** and which are consumed in fixed proportions and so aggregate output can be thought of as a composite commodity.

   The good is a flow commodity measured per unit of time. Be careful to distinguish between the various output, expenditure, and income measures - such as GDP, GDE, and GDI – all of which are flows per unit of time and the various stock magnitudes such as the various measures of the money supply and wealth – all of which can be measured

at any instant of time. In particular be very careful not to use the word money when you mean income.

3. There are no inventories and so production is assumed to be sold immediately that it is produced. We are doing QCS and so the model is **always** in equilibrium and, strictly speaking, we have nothing to say about out of equilibrium situations. The disequilibrium adjustment stories that we told you in Econ 207 are strictly meaningless in the context of QCS models. Inventories *can* be included in macro economic models but they introduce complicated dynamics, which we have neither the time nor the technique to handle here.

4. Order books are held constant and so firms respond to changes in demand by changing output not by delaying or speeding up delivery of their products.

5. Firms react instantaneously to changes in demand by adjusting their outputs. In our macro models in Econ 208 aggregate supply, AS, is a purely passive variable that increases and decreases exactly in line with changes in aggregate demand, AD. The three models we will construct do not attempt to explain the supply side of the economy – there is no labor market and only an implicit production function.

6. All profits are distributed to households as dividends that are, of course, paid in kind. $GDP_{MP} = GDE_{MP} = GDI_{FC.}$

7. There are no financial assets and no monetary sector to worry about. Our models are barter economies in which all transactions take place using the single produced good.

8. The model is short-run and so saving does not appreciably change the stock of real assets held by consumers and investment does not change potential output and there is no change in the size of the labor force.

9. There are enough underutilized resources for firms to be able to expand output whenever AD increases.

10. Initially we will be modeling a closed model although we will open the economy to foreign trade when we put model three together.

11. We will assume that the AD and AS functions are linear, that the vertical intercept of the AD curve lies above the vertical intercept of the AS curve (which is an identity function with zero intercept and a slope of one because AS = GDP_{MP} = GDI_{FC} = Y), and that the slope of the AD curve is less than the slope of the AS curve (=1).

You do **not** have to commit these assumptions to memory.

ECONOMIC THEORY 8

MACROECONOMIC MODELS

INTRODUCTION

1. The income determination models that we are now going to construct and analyze are, in many ways, simpler than the supply and demand models that we have been studying in previous sections of the Manual. Actually, although we will discuss three models, it is important to realize that the first two models are really just special cases of our final macro model (Model 3). Some textbooks, and some 207 instructors, go straight to Model 3 without bothering to build up that model in stages, as we will. What you will discover is that only one part of the model -- the AD function -- changes as we move from Model 1, to Model 2, and finally to the complete system, Model 3. This means that you need only learn one AS curve, etc., as you move from one model to the next.

2. There are other nice properties of the macro analysis that make it easier to do than supply and demand analysis. For example, when we are doing our QCS exercises we will only have one curve, the AD curve, to shift. And our macro models do not have perverse cases like Giffen goods and negatively sloped (aggregate) supply curves to make our lives complicated. Further because AS = AD = Y in equilibrium we only need to solve for one endogenous variable, Y, because when we know Y we also know AS and AD.

However, you need to be aware that students in earlier 208 classes have often found the macro part of the course more difficult than the supply and demand section. Some of this difficulty may simply reflect the fact that we do not have direct intuition about macro behavior in the way that most of us seem to understand the motivations of microeconomic agents. Another difficulty may be that many of you did 207 without formulating the relationships algebraically. And, finally, some students seem to find the algebraic AD equation difficult to understand and remember – it seems to be easy to get caught up in the algebraic foliage, and to forget the underlying economic tree.

3. You need to be completely familiar with the assumptions laid out in ET 7. This model is a **very** stylized version of an industrialized economy like the US that produced $10t of output in 2000. In particular you must keep reminding yourselves that this is a *barter* economy that does not use money, and that income, Y, (a *flow* variable) is **not** the same as the money *stock*, and that there are no monetary policy variables available for stabilization purposes in these models.

NATIONAL INCOME ACCOUNTING

1. Gross Domestic Product at market prices (GDP_{MP}) is the value of all **final** goods and services produced within the geographical boundaries of the country within one year. $GDP_{MP} = Y$ is the product of real output, y, and the aggregate price level, P. In our model P is a constant and is set equal to 1 so that y = Y, i.e. real and nominal GDP are identical to one another. We interpret $GDP_{MP} = Y = y$ as a

measure of aggregate economic output, which we refer to as Aggregate Supply, AS.

2. Gross Domestic Expenditure at market prices (GDE_{MP}) is the value of total expenditure by households, (consumption expenditures (C)), firms (gross investment on capital goods and services (I)), the government sector(s) (G)), and net foreign expenditures (exports (X) minus imports (M)). So GDE_{MP} can be thought of as measuring Aggregate Demand (AD) for goods and services, i.e. $AD = C + I + G + X - M$.

3. Gross Domestic Income at factor cost (GDI_{FC}) is the sum of all payments to factors of production, i.e. $GDI_{FC} = W+D+R+\Pi$ where W is wages and salaries paid to labor, D is dividends and interest paid to capital, R is rents paid to land and owners of natural resources, and Π is a residual (which can be negative) and is received by the entrepreneurs. There are no indirect taxes or subsidies and no net transfers to foreigners. So $GDI_{FC} = Y$ can be thought of as gross national income. In practice we will ignore depreciation (Capital Consumption Allowances) and proceed on the assumption that gross and net magnitudes are equal, i.e. $GDP_{MP} = NNP_{MP}$ etc.

4. In principle (with no measurement errors) $GDP_{MP} = GDE_{MP} = GDI_{FC}$, i.e. our three measures of aggregate economic activity should be equal. Hence AD = AS = Y from a measurement point of view, and they will also always be equal to one another in equilibrium -- we are doing QCS.

5. The only other concept that you need to be familiar with from national income accounting is disposable income, $Y^D = Y - T + TR$ where TR is government transfer payments which we