MATHEMATICAL REVIEW 4

SIMULTANEOUS LINEAR EQUATIONS

1. NUMBERS

Say $y_1 = f(x) = 2 + 3x$

$y_2 = g(x) = 12 - 2x$

$f(x) = g(x).$

Then $2 + 3x^e = 12 - 2x^e$

therefore $x^e = 2$

and $y^e = 2 + 3(x^e)$

therefore $y^e = 8.$

[Figure 1 goes here.]

2. SOLUTIONS

The solution set, S, of a set of two simultaneous equations is the set of x-y values that simultaneously satisfy the equations and therefore are coordinates on both curves, i.e.

$$S = \{(x,y): f(x)=g(x)\}.$$

In our numerical case the two linear equations in two unknowns (variables) possess a **unique** solution because they have different intercepts and slopes. There will be **no solutions** at all if the equations have *different intercepts* but the *same slopes* (in which case the lines are *parallel*), and there will be an **infinity of solutions** if the equations have *identical intercepts and slopes* (in which case the lines are *coincident*). If the equations possess at least one solution then they are said to be *consistent,* otherwise the equations are referred to as being *inconsistent*.

slope	y-intercept	nature	# solutions	type
same	same	coincide	infinite	consistent
same	different	parallel	none	inconsistent
different	different	intersect at exactly one point	unique	consistent

[Figure 2 goes here.]

3. ALGEBRA

If	$y = f(x) = a + bx$ $y = g(x) = c + dx$ $f(x) = g(x) \quad \text{where} \quad b \neq d$

then $\quad a + b\,x^e = c + d\,x^e$

hence $\quad b\,x^e - d\,x^e = c - a$

or $\quad d\,x^e - b\,x^e = a - c$

therefore $(d-b)\, x^e = a - c$

and $$x^e = \frac{a-c}{d-b} \qquad \text{(since } d-b \neq 0\text{).}$$

Further $$y^e = a + b\, x^e$$

$$= a + b\frac{(a-c)}{d-b}$$

$$= a\frac{(d-b)}{d-b} + b\frac{(a-c)}{d-b}$$

$$= \frac{ad-ab}{d-b} + \frac{ba-bc}{d-b}$$

$$y^e = \frac{ad-bc}{d-b}.$$

Note that there is no necessity for x^e and/or y^e to be positive.

[Figure 3 goes here.]

(Interpret the **three** cases specified in 2 above in terms of the algebra; i.e., when will we get *no solution*, *an infinity of solutions*, *a unique solution*?).

4. INTERPRETING THE SOLUTIONS

Assume that we start on the vertical axis where x is equal to zero. Then the numerator of x^e is either zero because $a=c$ or there is a positive gap between the two vertical intercepts with $a>c$ or $a<c$. If

a=c then the graphs of the two functions intersect at that point and $x^e = 0$. But if a>c or a<c then the graphs do not intersect on the vertical axis and so x^e will not be zero. If a>c then this gap must be eliminated if the two graphs are to intersect and the rate at which this gap is being eliminated is given by the denominator of the x^e equation, d-b the difference between the slopes of the two graphs. If d>0 and b<0 then the two graphs will converge somewhere between a and c and there will be a corresponding equilibrium value for x. If the gap between a and c is very large and the difference between the slopes is small then we will have to move a long way to the right in order to reach the point of intersection between the two graphs and x^e will be large, whereas if the gap between a and c is small and the difference between the two slopes is large then it will require only a small change in x to arrive at the intersection point of the two graphs and so x^e will be small. (If the slopes of the two graphs are equal then d = b which means that the graphs are parallel and d - b = 0 which means that the equation for x^e has no solution because division by zero is not a defined mathematical operation.) You must study the possible configurations of a, b, c, and d and determine how each one leads to a different set of equilibrium values for x and y. Note that mathematically we are not confined to strictly positive solution values for the two dependent variables. Notice also that the numerator of the equation for y^e is the distance between the horizontal intercepts of the graphs of the two functions, that is $ad - bc > 0 \Leftrightarrow (-a/b) - (-c/d) > 0$.

5. LINES AND PLANES IN 3-SPACE

One of the skills you need to acquire in Econ 208 is the ability to **generalize** a result. You should be able to convince yourselves that the following results hold for **three** simultaneous linear equations in **three** unknowns, where the graphs of the functions are **two dimensional planes** in a **three dimensional space**. There are five possible outcomes:

(a) No solutions (3 parallel *planes*).

(b) No solutions (2 of the planes are parallel).

(c) Infinitely many solutions (3 coincident planes).

(d) Infinitely many solutions (3 planes intersecting in a line).

(e) One solution (3 planes intersecting at a point).

Figure 1

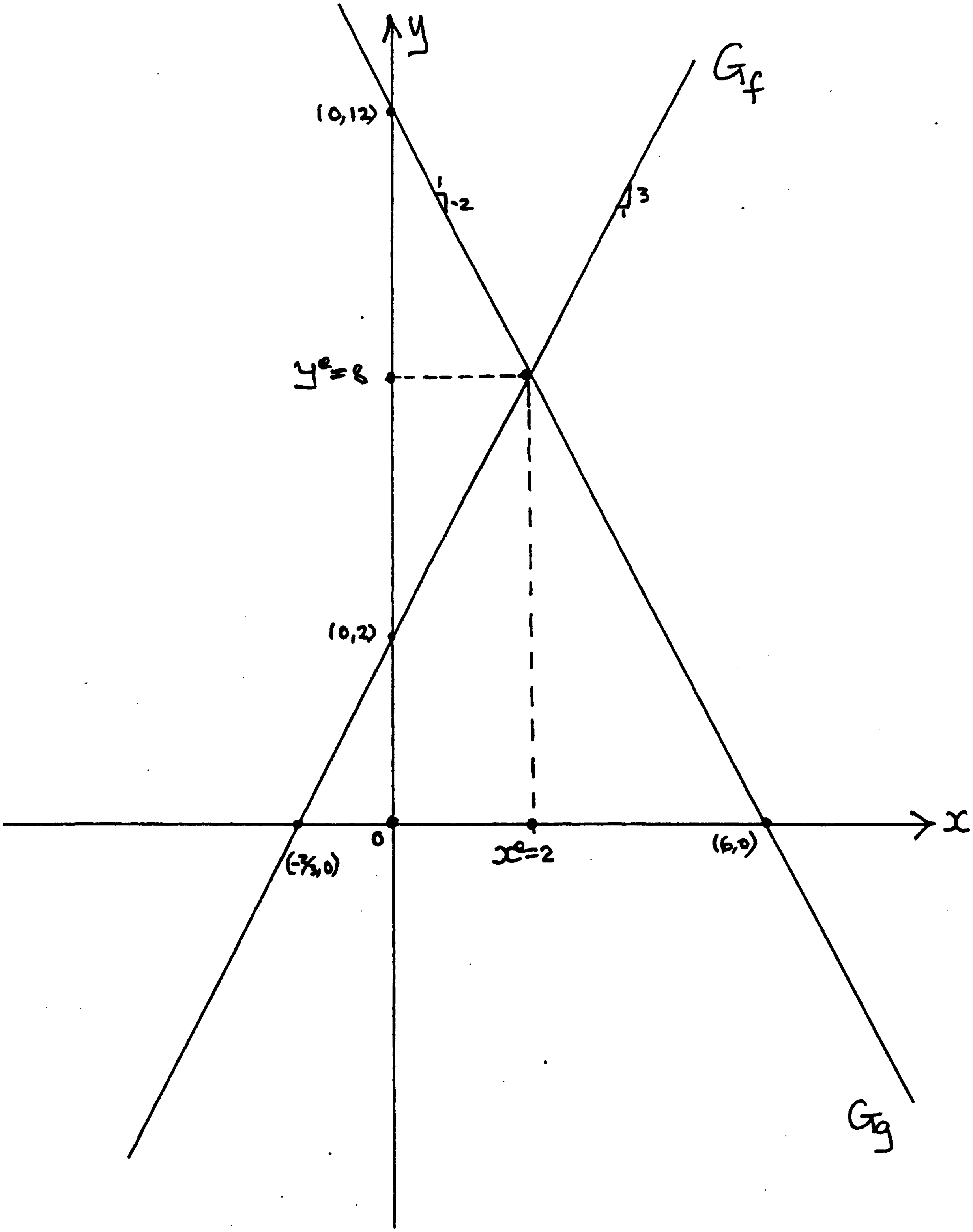

Figure 2

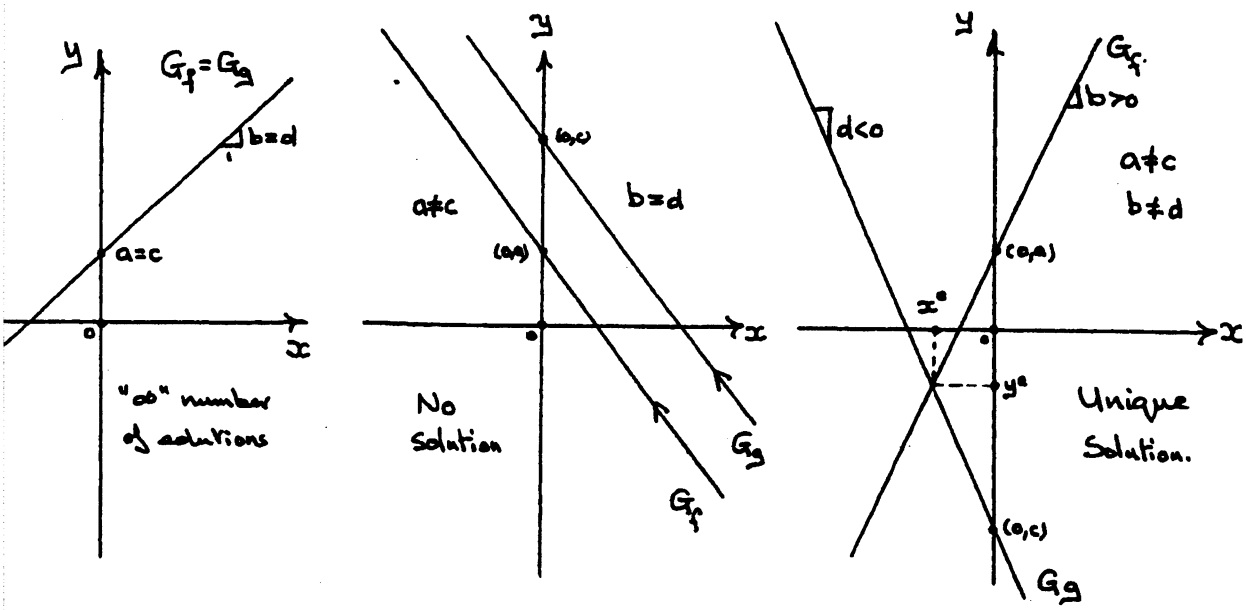

Figure 3