WE WILL *NOT* COVER THIS MATERIAL THIS QUARTER ALTHOUGH YOU MAY WANT TO READ THROUGH IT IF YOU ARE PLANNING TO TAKE ECON 306 SOON

ECONOMIC THEORY 3B

STATIC STABILITY ANALYSIS

1. Qualitative comparative static (QCS) is a powerful analytical tool but you must understand its limitations if you are to use it safely. We usually assume that demand curves have negative slopes and that supply curves have positive slopes. However, as we have seen in ET 3A and in class, we may wish to investigate unusual configurations of supply and demand curves. The Giffen good (GG) is a case in point. At least for "low" prices the demand curve may be positively sloped, and we also saw that inferior goods may have vertical demand curves at "low" prices. And while the short run supply curve will be positively sloped as soon as diminishing returns to the variable input sets in, in the long run it is perfectly reasonable to assume that the supply curve will be horizontal in the case of constant returns to scale and positively sloped in the face of increasing returns to scale -- a favorite topic of economists during the last fifteen years or so. Further, in asset markets -- for example the stock market or the foreign exchange market -- "expectational" effects may lead to perversely sloped demand and supply curves as transactors anticipate further price falls or increases, and markets exhibit herding or momentum effects. And, of course, in Econ 206 you considered vertical and horizontal demand and supply curves when studying elasticity of demand and supply. By now you are aware that demand and supply curves are unlikely to exhibit perverse behavior for all

prices but since our theoretical supply and demand diagrams have no scales on the axes we will often draw the demand and supply curves with the non-traditional slopes over the whole range of prices shown in our diagrams.

2. To do QCS correctly you need to always keep in mind that in Econ 208 you are doing economics, not geometry or algebra or calculus, although we make heavy use of these mathematical tools in the course. Therefore, your analysis must make economic sense and you must beware of mindless curve shifting or symbol manipulation. You must always ask, when doing a piece of economic analysis, if the underlying logic of your model makes sense. Paul Samuelson showed us in "The Foundations of Economic Analysis" – his Ph.D. thesis, largely written in 1937 when he was twenty two years old, although it did not see print (and that is an interesting story!) until 1947 – that QCS **only** gives sensible answers if the underlying model is statically stable. It was this insight that was cited by the Nobel Prize committee when it awarded Samuelson the first Nobel Prize in economics won by an individual (and only the second ever awarded).

3. Static stability analysis is a technique that economists have borrowed, as so much else, from physics – specifically from mechanics. Stability analysis really requires us to do true dynamics, but that means that you have to be able to handle differential or difference equations, which would take us well beyond the mathematical borders of Econ 208. (A&L give a nice treatment of static stability analysis in Ch.3.5 – and you may want to read sections 3.6 and 3.7 too. Their chapters 12 -14 cover true dynamics.) We will proceed heuristically and rely largely on economic intuition in our analysis -- and a considerable amount of "hand waving"!

4. Before we proceed, it is important for you to understand that **in this discussion** of the stability of our supply and demand models **we will violate** the dictum that I laid down when we first began to study QCS seriously, i.e. *the rule that statements made about the model when it is not in equilibrium are technically nonsense, since QCS deals only with equilibrium states*. However, we now need to tell economic stories about what happens when prices are not equilibrium prices. To do this formally would, as I have already noted, require us to push our mathematical envelope further than you would probably feel comfortable with, and so we will leave this fun topic for some other economics course.

5. The crucial idea that you must internalize is this. **If there is excess demand** *at a given price*, **then market forces will cause *the price to <u>rise</u>*,** and **if there is excess supply** *at a given price* **then market forces will cause *the price to <u>fall</u>*.**

6. Consider the standard supply and demand diagram illustrated in Figure 1. At the price, P^e, the quantity demanded is Q_e^D, and the quantity supplied is, Q_e^S. But $Q_e^D = Q_e^S$ and so the market is in equilibrium. Therefore, the quantity transacted at P^e, i.e. Q^e, is equal to the quantity demanded, Q_e^D, and the quantity supplied, Q_e^S. If the market is in equilibrium then there are no unsatisfied transactors, i.e. every household that wishes to buy their utility maximizing quantity at P^e can find a seller, and every profit maximizing firm that wishes to sell at P^e can find a buyer. The forces of supply and demand are in balance and there is nothing to cause the price to change. Therefore, once the market settles down at (Q^e, P^e) it will stay there until some force, a violation of the ceteris paribus assumption causes a state of disequilibrium to arise.

7. The (Q^e, P^e) configuration of the model is said to be statically stable because this configuration of the supply and demand model has *a built-in self-correcting property*. Say there is some random shock to the system; not a change in one of the exogenous variables because that would shift the demand or supply curve, but something like a strike, a tornado, or an earthquake that suddenly shifts the price to P_1. At the "high" (above equilibrium) price P_1 the quantity demanded by households is only Q_1^D, whereas the "high" (above equilibrium) price, P_1 stimulates the profit maximizing firms to produce the quantity supplied, Q_1^S. But $Q_1^S > Q_1^D$ and so there is excess supply, a surplus, at P_1 equal to $E^S (P_1) = Q_1^S - Q_1^D$. The market is no longer in equilibrium and there are unsatisfied transactors, i.e. firms who want to produce and sell Q_1^S but who cannot find buyers at the price P_1 because households are only demanding Q_1^D.

Whenever there is excess supply we will assume that some firms who cannot sell as much as they wish to at the current price, P_1, will start to bid the price of the good down in the expectation that they will be able to find more buyers at a lower price. Since the demand curve is downward sloping and the supply curve is upward sloping the excess supply will be smaller at the lower price, but so long as the price remains above the equilibrium level there will continue to be some excess supply and the price will continue to fall until we return to equilibrium at P^e. (This story about how markets automatically adjust to random shocks is one of the desirable features of market systems that economists like to emphasize. But you need to remember that this is just a story, not a mathematical proof, and that *real world markets may not behave in this accommodating fashion*.)

Now there is nothing about special about P_1 except that it is above the equilibrium price, P^e. Consequently we can assume that at *any* price above the equilibrium price, price will fall.

However, our analysis leaves much to be desired since our model is mute about what happens when the market is not in equilibrium and so we cannot say anything about how P (or Q) adjusts nor can we say how long the adjustment process will take. Therefore in Figure 2 we have shown the sudden increase in the price from P^e to P_1 taking place at the arbitrary point of time, t_0, and market price being restored to its initial equilibrium at the arbitrary point of time, t_1. The diagram does not show the path that P takes between t_0 and t_1 and we do not know how long the interval t_1-t_0 is.

8. Now let us inflict a second random shock on the system (at time t_2) that causes the price to fall to P_2. At P_2 it is clear that we have excess demand (a shortage) because households want to purchase Q_2^D at P_2 whereas firms will only supply Q_2^S and so we have $E^D (P_2) = Q_2^D - Q_2^S$. We now have unsatisfied consumers.

We will assume that whenever there is excess demand that some of the households will attempt to increase their purchases by bidding the price upwards. Because the demand curve is downward sloping and the supply curve is upward sloping, the higher price will be associated with a smaller excess demand for the good, but, so long as there is any excess demand whatsoever households will continue to bid prices up until we return to equilibrium at P^e.

Once again we cannot say anything about the time paths of P or Q between t_2 and t_3, nor do we know how long the time interval t_3 - t_2 is. So in Figure 3 we show the sudden drop in P at t_2 and the price restored to its original level at t_3 but we leave a gap between t_2 and t_3 to reflect our ignorance of the disequilibrium behavior of the model.

9. The example that physicists use to illustrate this homeostatic (self-correcting) property of a statically stable system is a ball bearing and a metal bowl. In Figure 4a we see the stable configuration. The ball bearing starts at equilibrium at the bottom of the bowl. Then we disturb the system – the forefinger random shock! – by shifting the ball bearing up the side of the bowl (but not so far that it goes over the lip of the bowl). If we then let go the force of gravity will cause the ball to move back down the side of the bowl. It is very unlikely that the ball bearing will simply move back to its original position at the bottom of the bowl. Probably the ball bearing will overshoot and move up the opposite side of the bowl until the force of gravity tugs it back down again. In addition there will be friction between the ball bearing and the surface of the bowl and so the ball bearing will, ultimately end up once again in its original position at the bottom of the bowl, as the forces acting on the ball finally cancel out and it reaches a state of rest – equilibrium.

   Physicists are able to describe this simple system with a set of differential equations and are able to measure the exact initial displacement of the ball bearing, accurately measure the force of gravity, and the effects of friction and of air pressure on the ball bearing. As a consequence they are able to make some rather precise statements about the behavior of the ball bearing. And, of course, they go to great pains to ensure that no other forces were disturbing the system, such as something shifting the table on which the bowl rests. In other words physicists would try to control everything so that they could be sure that the forces acting on the ball bearing were the ones that they were interested in. We economists do not (usually) have such an easy time of it, and our assumption of ceteris paribus might not hold if we were looking at a real market. On the other hand physicists would have much more problems analyzing the behavior of an autumn leaf falling from a tree on a gusty day, so we economists do not have to feel all that modest about our abilities – actually a modest economist is something of an oxymoron!

10. Figure 4b shows a set up that is not stable and self-correcting. We start from equilibrium with the ball bearing resting at the apex of the inverted bowl. Now if the ball bearing is subject to the slightest disturbance – a puff of breath rather than that forceful digit – then the ball bearing will slide down the side of the bottom of the bowl and will end up on the table – perhaps on the floor. In this case there are no forces to return the ball bearing to its original equilibrium state.

    Figure 5 – which is Figure 4 with the labels on the curves reversed – illustrates an unstable equilibrium. Now at P_1 we have excess demand ($E^D(P_1) = Q_1^D - Q_1^S$) and so, according to our rule about what happens when there is excess demand and unsatisfied buyers, households will start to bid up the price of X. But, as you can see from the diagram, this makes the situation worse because the price increase will cause the excess demand to get worse, not better, and so the price will go off to infinity.

    In the case where the price shock moves us to P_2 there will be excess supply. According to our hypothesis the firms who cannot sell their profit maximizing quantity at P_2 will start to cut prices, but in this case the fall in the price of X simply leads to even more excess supply and so the price will continue to fall until it finally drops to zero. So the situation in Figure 5 is like the inverted bowl. The initial equilibrium may exist but any shock to the system leads price and quantity to move further and further away from equilibrium.

12. In Figure 6 we show a stable situation in which there is excess supply above the initial equilibrium price, P_0^e, and excess demand below the equilibrium price. In this case we are safe to do our standard QCS exercises. For example, if there is a change in an exogenous supply side variable that causes the supply curve to shift to S^* then there will be excess supply at the original price, i.e. $E^S(P_0^e) = Q^{S^*} - Q^{D^*}$ and so there will be some firms who cannot sell there profit maximizing output at the price P_0^e. These firms will bid down the price which will cause the excess supply to decrease, but the price will continue to fall so long as there is any excess supply whatsoever, and we will therefore end up in equilibrium at (Q_1^e, P_1^e). You should do the cases in which the supply curve shifts to the left and the demand curve shifts to the right or left to convince yourself that in each of these cases the model will move from one equilibrium to the next in conformity with economic logic.

13. In Figure 7 we have another stable situation. We know this because at **any** price, say P_1, above the initial equilibrium price, P_0^e, there is excess supply ($E^S = Q_1^S(P_1) - Q_1^D(P_1)$) and therefore price will fall, and at **any** price, say P_2, below the initial equilibrium price, P_0^e, there is excess demand ($E^D = Q_2^S(P_2) - Q_2^D(P_2)$) and therefore price will rise. Therefore, the initial equilibrium at (Q_0^e, P_0^e) – with the supply curve S – is a stable equilibrium and we can therefore safely do our

QCS exercises (even though the demand curve is positively sloped for all prices shown).

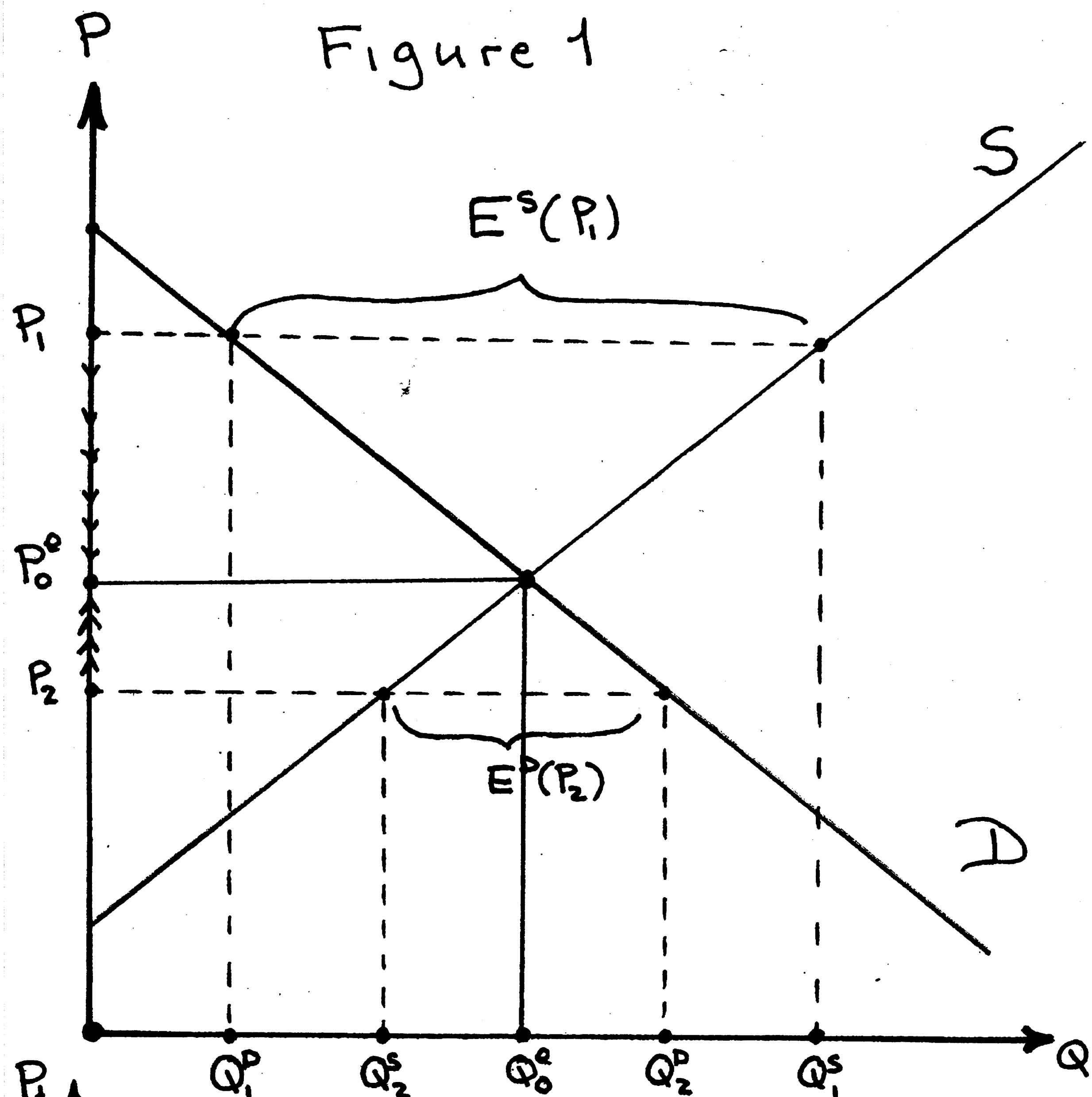
14. For example, we can shift the supply curve to the right from S to S^*. The increase in supply causes excess supply at the old equilibrium price, P_0^e, i.e. $E^S(P_0^e) = Q^{S^*}(P_0^e) - Q^{D^*}(P_0^e)$. Unsatisfied sellers will bid prices down until the excess supply is eliminated at the new equilibrium price, P_1^e. The fall in the price causes consumers to purchase less and we move down the given positively sloped demand curve to (Q_1^e, P_1^e).

The demand and supply configuration shown in Figure 8 does not allow us to do our QCS exercises because this configuration is clearly an unstable one. Consider any price, say P_1, above the initial equilibrium price established at P_0^e. At P_1 there is excess demand ($E_1^D = Q_1^D(P_1) - Q_1^S(P_1)$) and therefore unsatisfied buyers will bid the price up. But that will only make the excess demand worse because the gap between D and S is widening not narrowing. Ultimately the price will "go off to infinity".

Consider next any price below the initial the price equilibrium price established at P_0^e. At $P_{2,}$ for example, there is excess supply ($E_2^S = Q_2^S(P_2) - Q_2^D(P_2)$) and therefore unsatisfied sellers will bid the price down and away from equilibrium. The fall in the price will only make the excess supply worse because the gap between S and D is widening not narrowing. Ultimately the price will fall to zero and the good will no longer be produced.

Say we are at the initial equilibrium and that the supply curve shifts to the right from S to S^*. According to the diagram there exists a new equilibrium at (Q_1^e, P_1^e), with a higher price and more of the good transacted in each time period. But this equilibrium will never be reached because the shift of the supply curve causes excess supply at the original price ($E^{S*} = Q^{S*}(P_0^e) - Q_0^D(P_0^e)$) and unsatisfied suppliers will start to bid the price **down** which will move us away from the new "equilibrium". Indeed, once the price begins to fall it will continue to do so as the excess supply becomes larger, not smaller, and ultimately the price and quantity will end up both being zero.

Figure 1



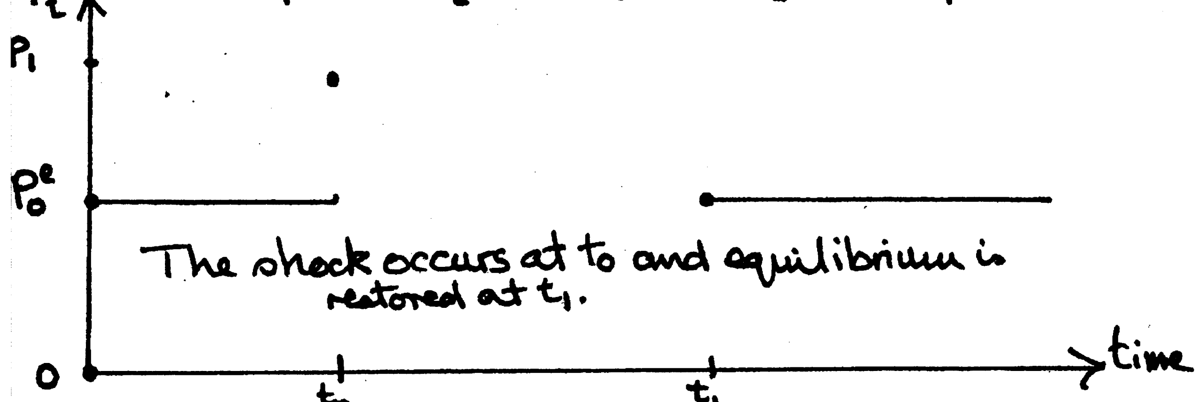


Figure 2

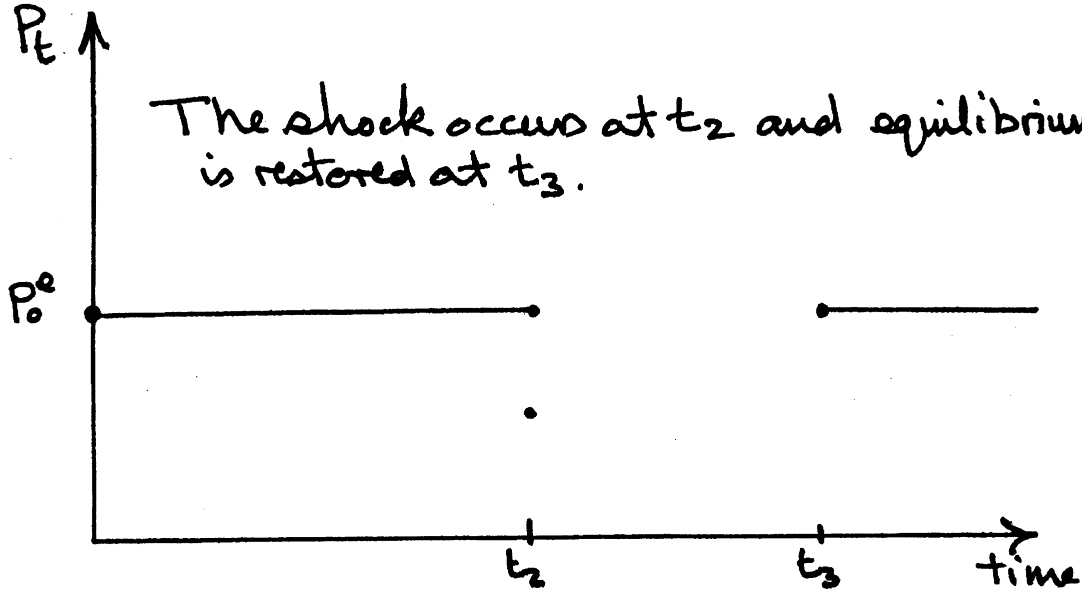
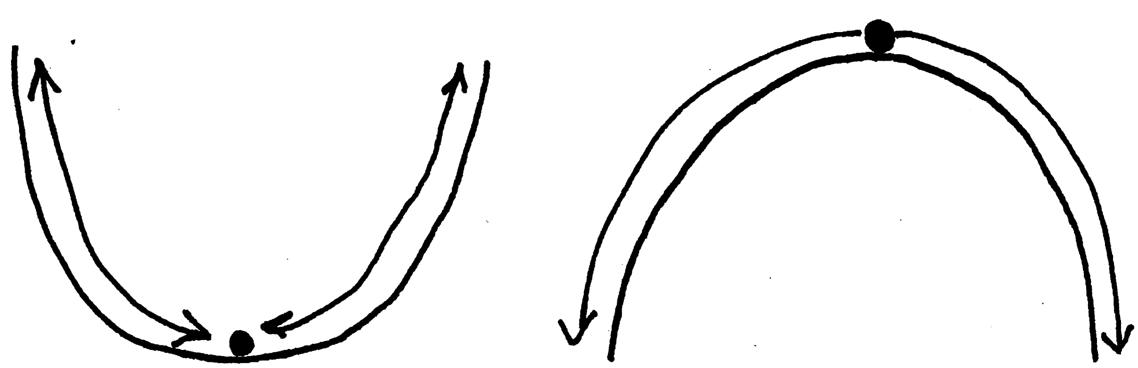


Figure 3

Figure 4



(a) (b)

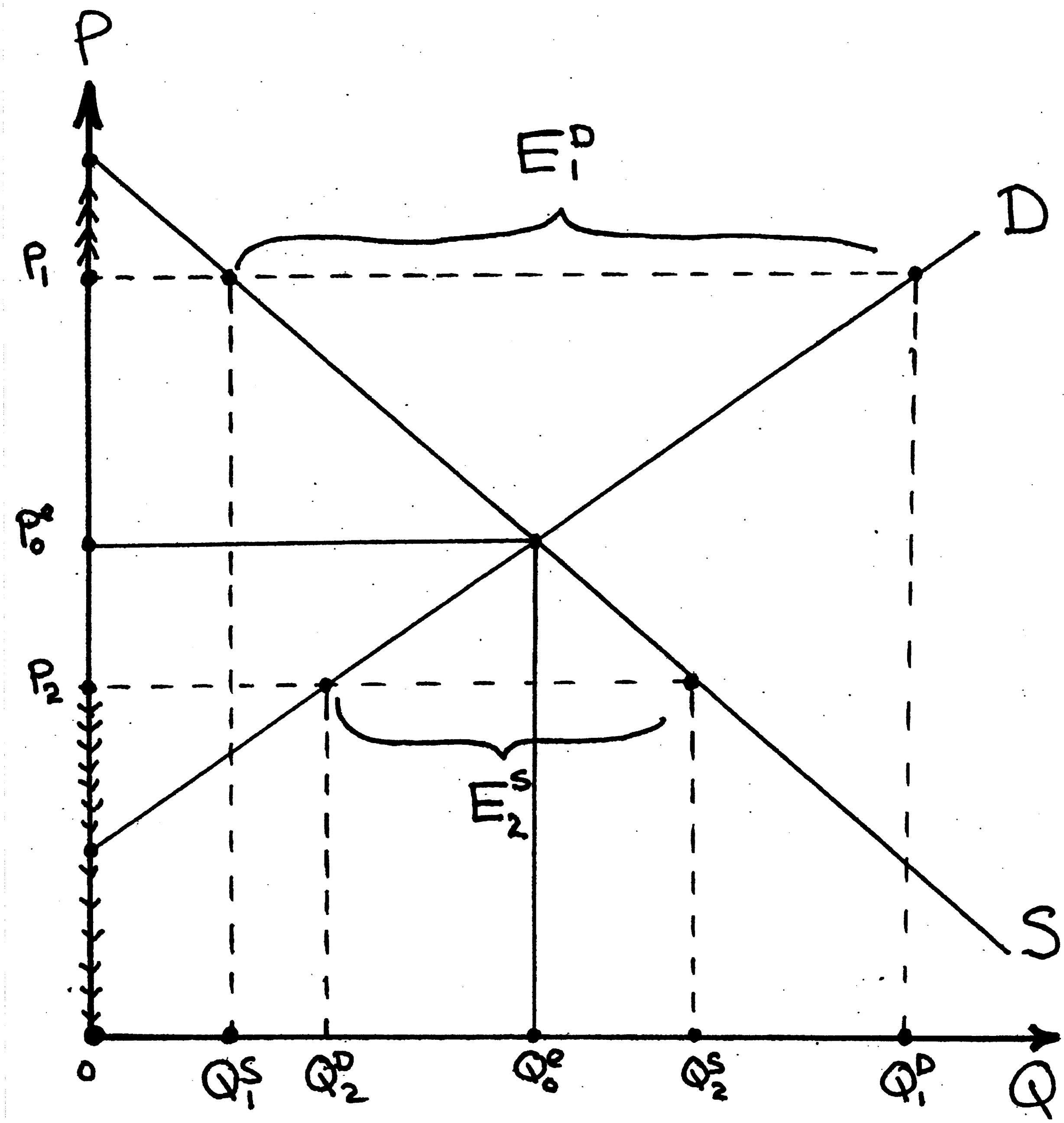


Figure 5

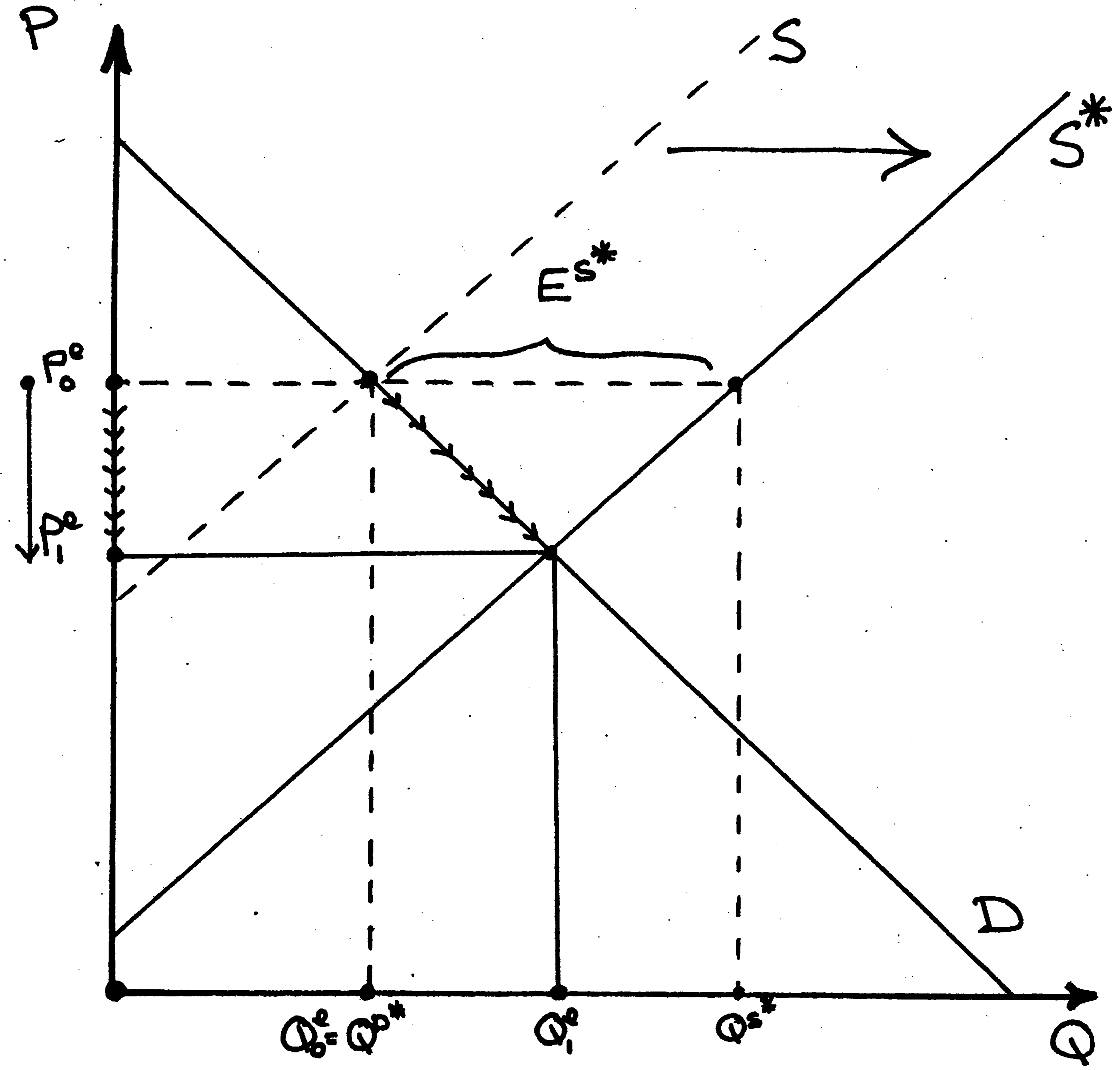


Figure 6

Stable equilibrium. The supply shock ($S \rightarrow S^*$) causes price to fall and there is a new equilibrium at (Q_1^e, P_1^e).

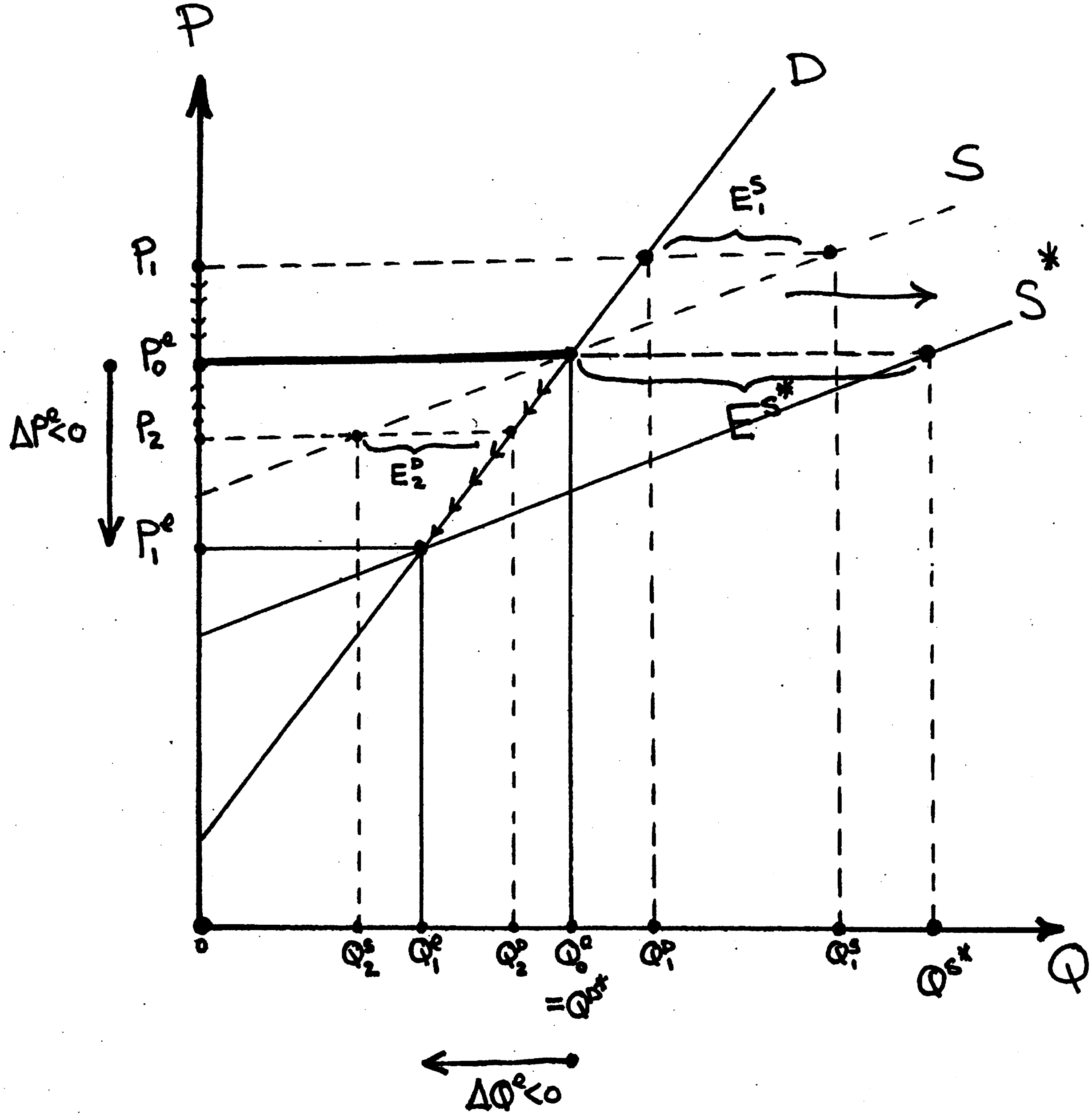


Figure 7.
Stable equilibrium. The supply shock ($S \rightarrow S^*$) causes excess supply E^{S^*} and the price falls to P_1^e.

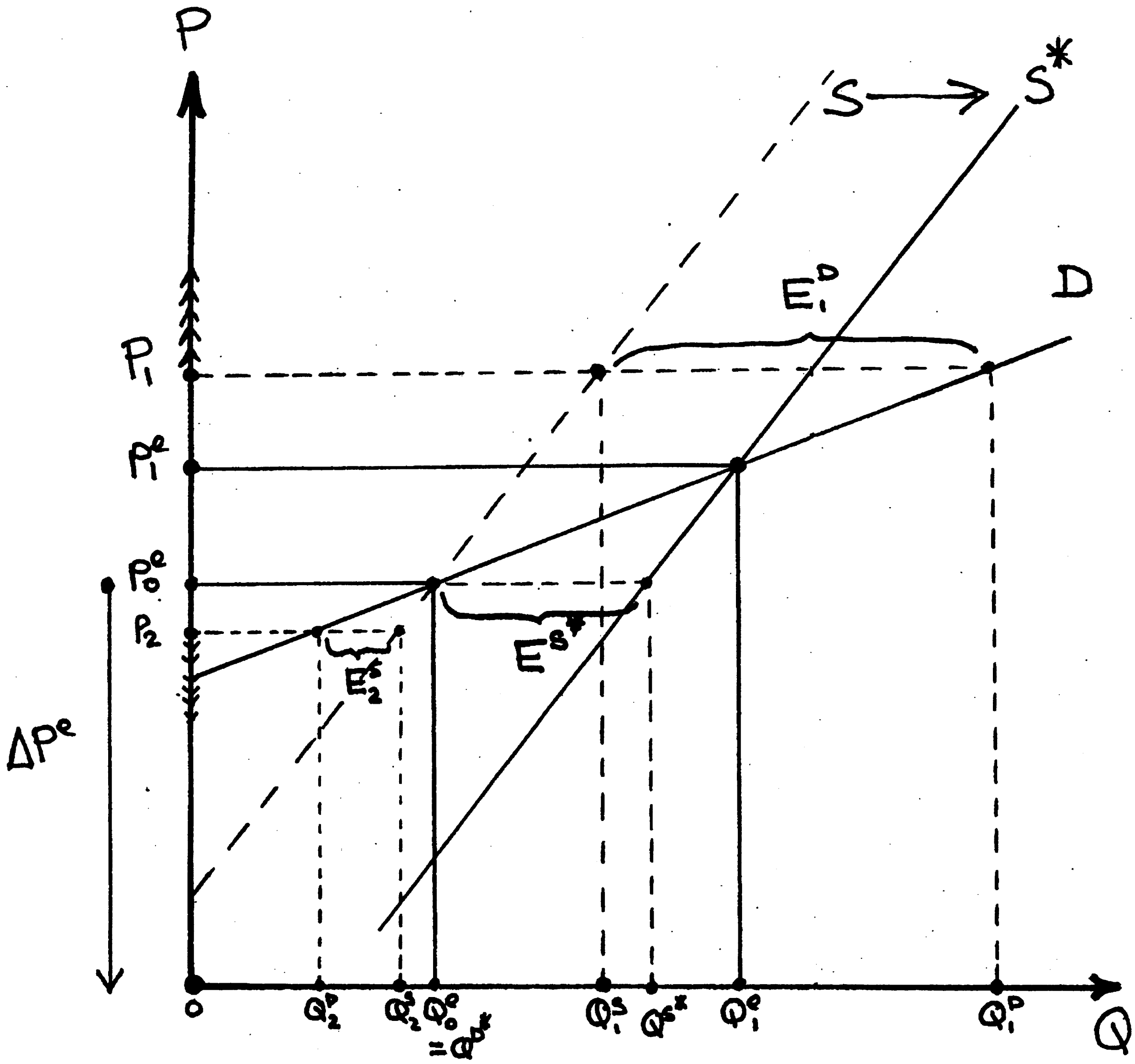


Figure 8

Unstable equilibrium. The supply shock ($S \to S^*$) causes P to fall to zero (Q_1^e, P_1^e) exists but will not be attained.